Cellular Automata

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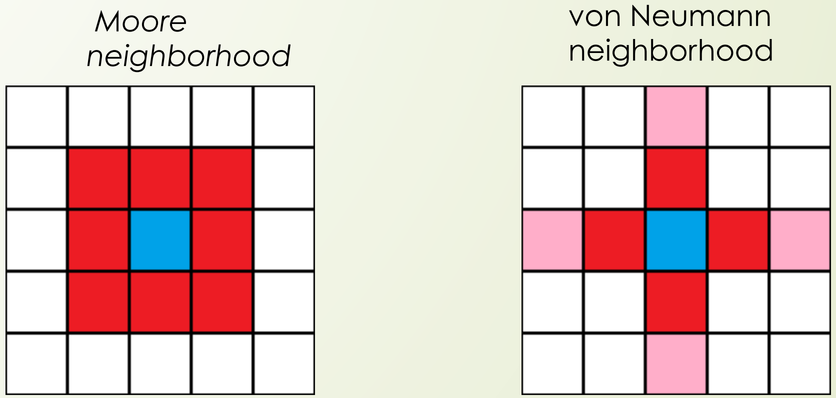
Outline

- What is a Cellular Automaton?
- Wolfram's Elementary CA
- Conway's Game of Life
- Applications and Summary

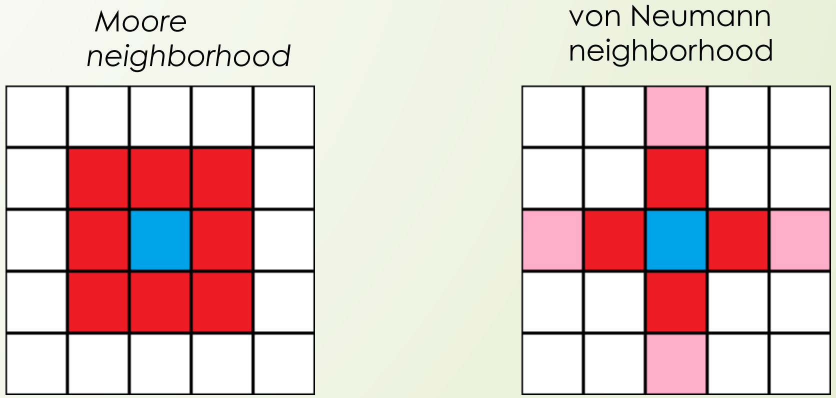
Cellular automaton

- A **discrete** model
- Regular grid of **cells**, each in one of a **finite number of states**
- The grid can be in any finite number of dimensions
- For each cell, a set of cells called its **neighborhood** is defined relative to the specified cell



Cellular automaton

- An initial state (time t=0) is selected by assigning a state for each cell
- A new generation is created (advancing t by 1), according to some fixed rule that determines the new state of each cell in terms of:
  - the current state of the cell
  - the states of the cells in its neighborhood
- Typically, the rule set is
  - the same for each cell
  - does not change over time
  - applied to the whole grid simultaneously



Background

Ulam

von Neumann

- Originally discovered in the 1940s by Stanislaw Ulam and John von Neumann
- Ulam was studying the growth of crystals and von Neumann was imagining a world of self-replicating robots
- Studied by some throughout the 1950s and 1960s
- **Conway's Game of Life** (1970), a two-dimensional cellular automaton, interest in the subject expanded beyond academia
- In the 1980s, Stephen Wolfram engaged in a systematic study of one-dimensional cellular automata (**elementary cellular automata**)
- Wolfram's research assistant Matthew Cook showed that one of these rules has a VERY cool and important property
- Wolfram published *A New Kind of Science* in 2002 and discusses how CA are not simply cool, but are relevant to the study of many fields in science, such as biology, chemistry, physics, **computer processors and cryptography**, and many more

Why the big interest???

- A complex system, generated from a very simple configuration
  - Is this even possible?

- Cellular automata can simulate a variety of real-world systems. There has been speculation that **CA may be able to model reality itself**, i.e. that the **universe could be viewed as a giant cellular automaton**.

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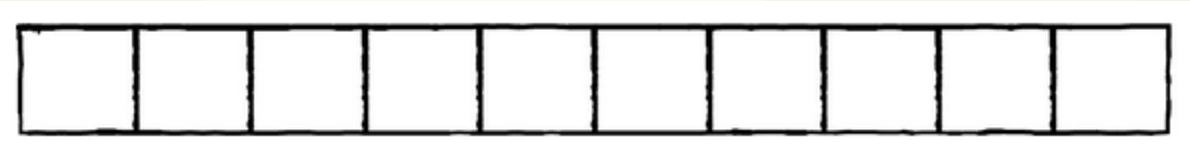
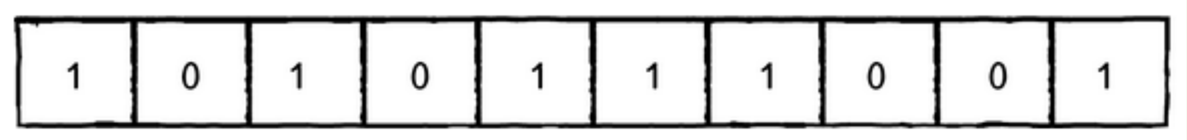
General One-Dimensional CA

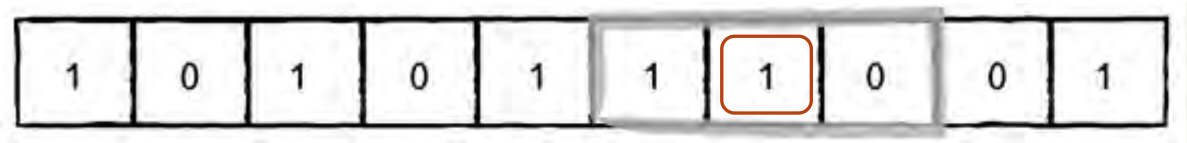
- Grid



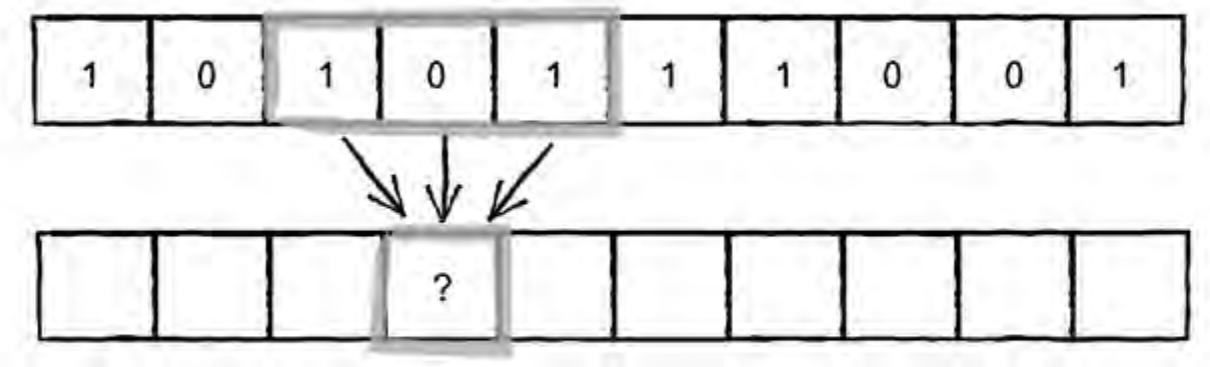
- Initial state



- Neighborhood



- Rules

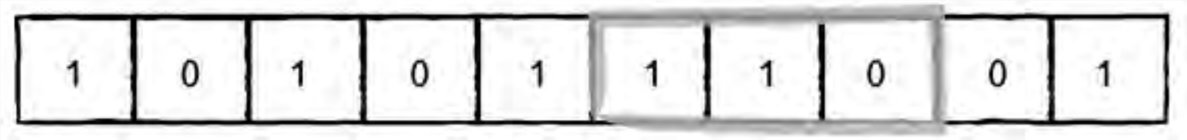


Average,
Sum
…

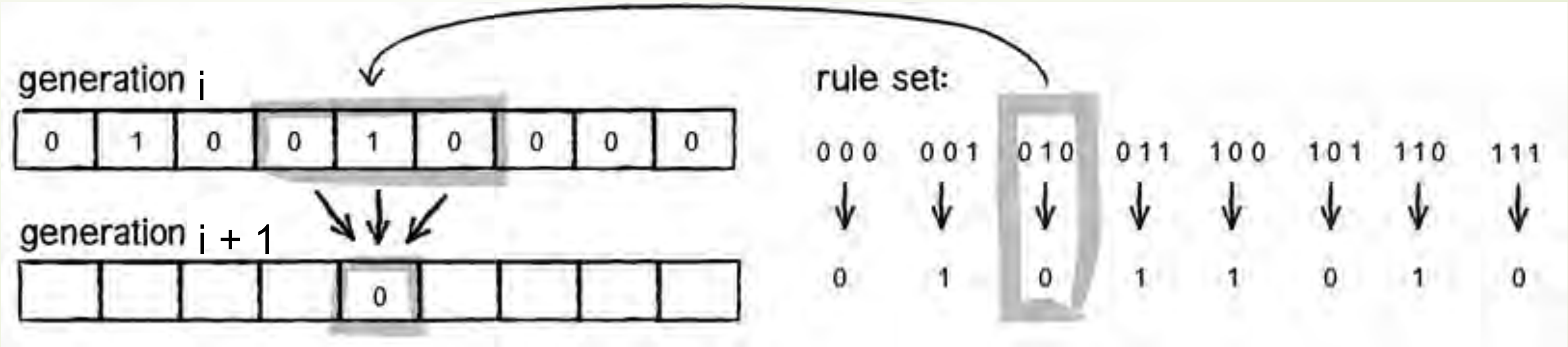
Wolfram's Elementary CA

- Simplest possible model
- One-dimensional cellular automaton
- The standard Wolfram model is to initialize the grid with all cells assigned with 0, except for the middle cell which is assigned with 1

- The neighborhood is the two immediate neighbors

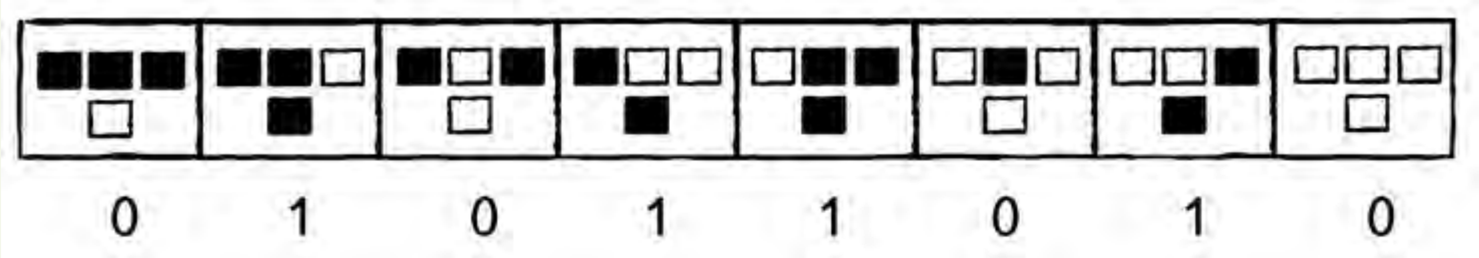


- Rules



Wolfram's Elementary CA - Demo

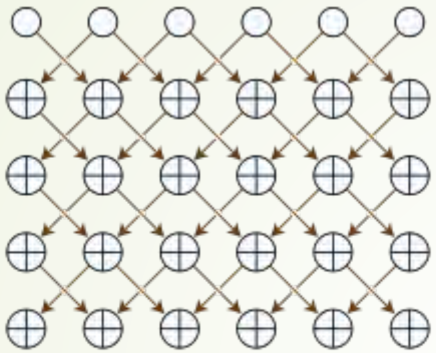
- In the demo we'll use this set of rules:

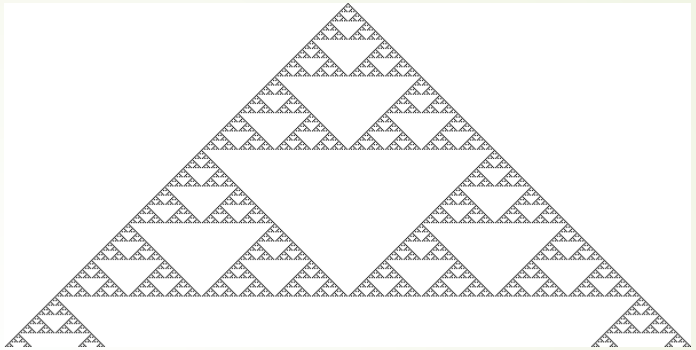


- These set of rules are called "Rule 90"

Demo

Wolfram's Elementary CA – Rule 90



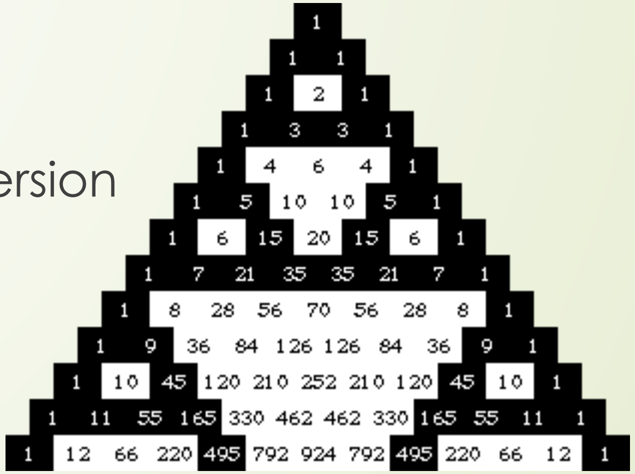


- Rules: XOR of the two neighboring values
- "the simplest non-trivial cellular automaton" (Martin, Odlyzko & Wolfram, '84)
- Interesting properties:
  - When started from a **random** initial configuration, its configuration remains **random** at each time step
  - Any configuration with only **finitely many nonzero cells** becomes a **replicator** that eventually **fills all of the cells with copies of itself**
  - ***Additive cellular automaton***: if two initial states are combined by computing the XOR of each their states, then their subsequent configurations will be combined in the same way

Rule 90 & Sierpiński triangle

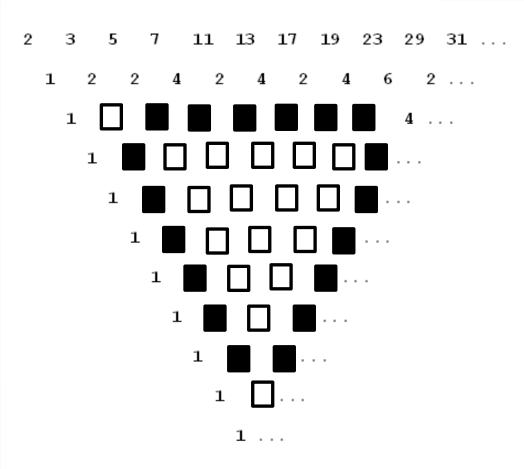
- The time-space diagram of Rule 90 has the appearance of the **Sierpiński triangle fractal** when responding to an initial state with a single nonzero cell

- Because the XOR rule is equivalent to modulo-2 addition, this generates the modulo-2 version of **Pascal's triangle**, which is a discrete version of the Sierpiński triangle



Rule 90 - Uses

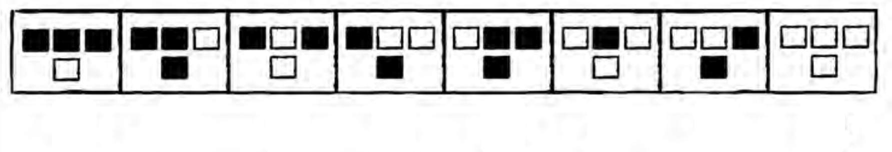
- Gilbreath's conjecture on the differences of consecutive prime numbers



  - Rule 90 controls the behavior of the parts of the rows that contain only the values 0 and 2
- From any initial configuration of Rule 90, one may form an acyclic graph in which every node has at most one outgoing edge

Wolfram's Elementary CA – Classes

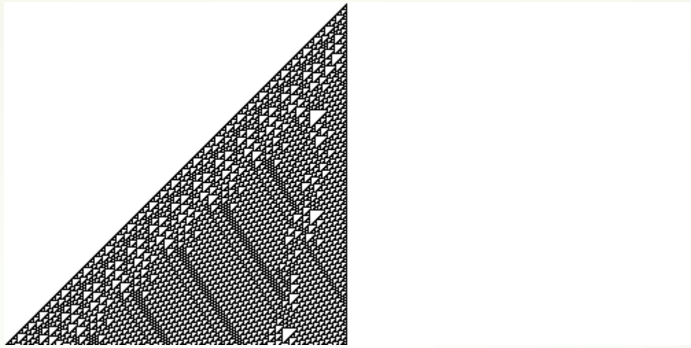
- There are only 256(= 2^8) different ways for a Wolfram Elementary CA can be defined!



- They can be categorized into 4 classifications:
  - CA which rapidly converge to a uniform state (rule 222)
  - CA which rapidly converge to a repetitive or stable state (rule 190)
  - CA which appear to remain in a random state (rule 30)
  - CA which form areas of repetitive or stable states, but also form structures that interact with each other in complicated ways – a complex system (rule 110)
    - These are thought to be computationally universal, or **capable of simulating a Turing machine**

Demo

Wolfram's Elementary CA – Rule 110



- From the 'Complexity' class
- Interesting behavior on the boundary between stability and chaos
- Rule 110 is known to be **Turing complete**. This implies that, any calculation or computer program can be simulated using this automaton (like λ–calculus)
- Rule 110 is arguably the simplest known Turing complete system
- This result therefore provides significant support for Wolfram's view that class 4 systems are inherently likely to be universal

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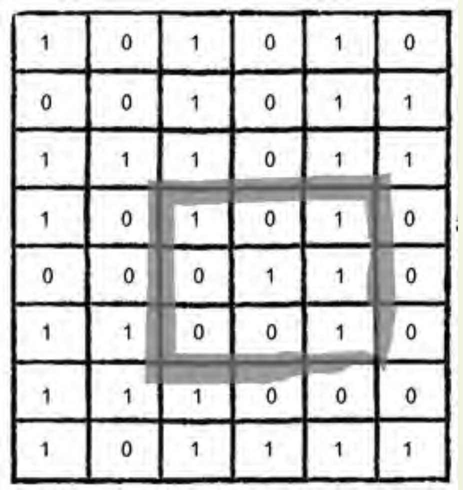
Conway's Game Of Life - History

- von Neumann attempted to find a hypothetical machine that could build copies of itself and succeeded when he found a mathematical model for such a machine with very complicated rules on a rectangular grid
- **The Game of Life** emerged as Conway's successful attempt to drastically simplify von Neumann's ideas and still achieve similar **“lifelike”** result
- Conway's Game of Life has the power of a **universal Turing machine**: that is, anything that can be computed algorithmically can be computed within the game
- The game made its first public appearance in the October 1970 issue of Scientific American, in Martin Gardner's "**Mathematical Games**" column

Conway's Game Of Life

- A two-dimensional CA
- Each cell is initialized with a random state: 0 or 1.
- In this case, 0 means dead and 1 means alive
- Each cell will have a bigger neighborhood
- With nine cells as the neighborhood, we have 9 bits, or 512 possible configurations - impractical to define an outcome for every single possibility (This is Moore neighborhood, the total number of automata possible would be 2^{2^9} , or 1.34×10^{154})



1	0	1	0	1	0
0	0	1	0	1	1
1	1	1	0	1	1
1	0	1	0	1	0
0	0	0	1	1	0
1	1	0	0	1	0
1	1	1	0	0	0
1	0	1	1	1	1

Conway's Game Of Life – The Rules

- A cell can die if
  - The cell is alive and has fewer than two live neighbors (loneliness)
  - The cell is alive and has more than three live neighbors (overpopulation)
- A cell can reborn if
  - The cell is dead and has exactly three live neighbors (reproduction)
- A cell remains in its current state if
  - The cell is alive and has two or three live neighbors
  - The cell is dead and has anything other than three live neighbors

Demo

Mathematical Games (1970)

The fantastic combinations of John Conway's new solitaire game "life"

- “This month we consider Conway's latest brainchild, a fantastic solitaire pastime he calls "life“”
- Conway's game belongs to a growing class of what are called **"simulation games"- games that resemble real-life processes** due to its analogies with the rise, fall and alternations of a society of living organisms
- The basic idea is to start with a simple configuration of organisms, one to a cell
- Then observe how it changes as you apply Conway's **"genetic laws" for births, deaths, and survivals**

Mathematical Games (1970)

The fantastic combinations of John Conway's new solitaire game "life"

- Conway chose his rules carefully, after a long period of experimentation, to meet three desiderata:
  - There should be no initial pattern for which there is a simple proof that the population can grow without limit.
  - There should be initial patterns that *apparently* do grow without limit.
  - There should be simple initial patterns that grow and change for a considerable period of time before coming to end in three possible ways: fading away completely (from overcrowding or becoming too sparse), settling into a stable configuration that remains unchanged thereafter, or entering an oscillating phase in which they repeat an endless cycle of two or more periods
- In brief, the rules should be such as to make the behavior of the population **unpredictable**

Mathematical Games (1970)

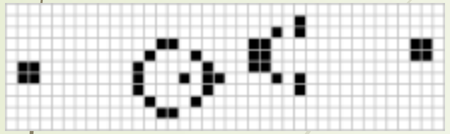
The fantastic combinations of John Conway's new solitaire game "life"

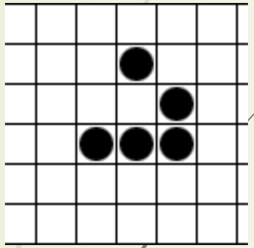
- You will find the population constantly undergoing unusual, sometimes beautiful and always unexpected change
- In a few cases the society eventually dies out
- **Most starting patterns** either reach stable figures that **cannot change** or patterns that **oscillate forever**
- Patterns with no initial symmetry tend to become symmetrical
- Once this happens the symmetry cannot be lost, although it may increase in richness

Conway's Conjecture

- Conway conjectures that no pattern can grow without limit
- This is probably the deepest and most difficult question posed by the game

Conway’s Conjecture - Disproved





- A **gun** is a pattern with a main part that repeats periodically, like an oscillator, and that also periodically emits spaceships
- A finite pattern is called a **spaceship** if it reappears after a certain number of generations in the same orientation but in a different position
- The **glider** is a pattern that travels across the board in GOL. Gliders are the smallest spaceships, and they travel diagonally.
- Bill Gosper discovered the first glider gun (and, so far, the smallest one found) in 1970, earning $50 from Conway
- This eventually led to the proof that Conway's Game of Life could function as a Turing machine

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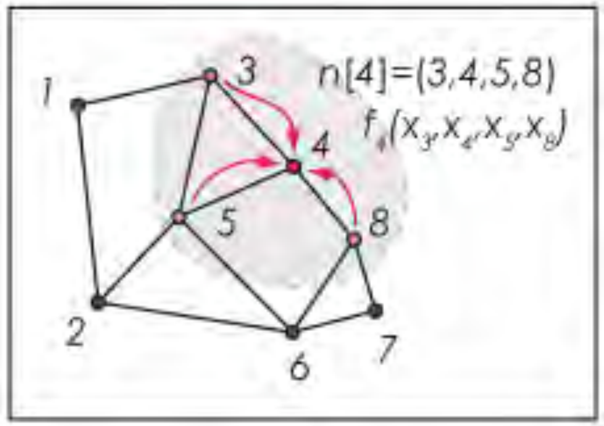
Graph Dynamical System (GDS)

- A Graph Dynamical System is a triple consisting of:
  - A graph Y with vertex set $v[Y]=\{1,2,\ldots,n\}$
  - For each vertex i there is:
    - a state $x_i \in K$
    - a Y-local function $F_i : K^n \rightarrow K^n$

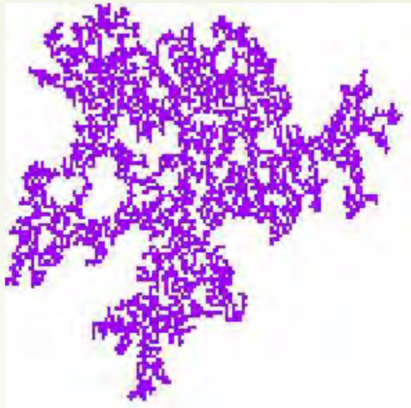
    $F_i(x=(x_1,\ldots,x_n))=(x_1,\ldots,x_{i-1},f_i(x[i]),x_{i+1},\ldots,x_n)$
  - An update scheme that governs how the maps F_i are assembled to a map $F: K^n \rightarrow K^n$
  - Typical choices of update schemes:
    - Parallel: Generalized Cellular Automata
    - Sequential: Sequential Dynamical Systems



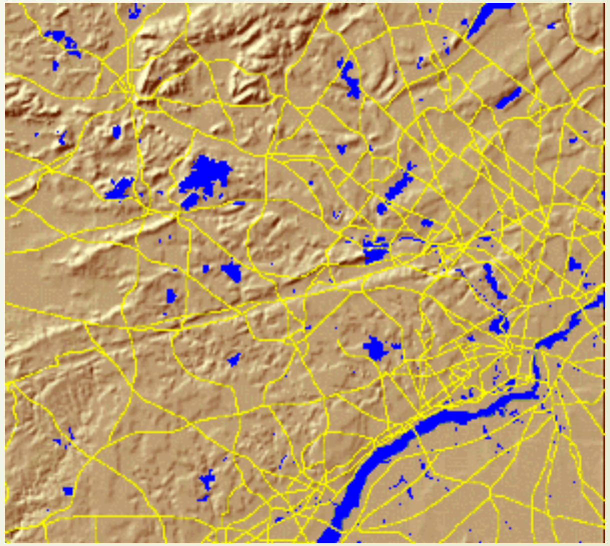
GDS Applications

- Dynamical processes on networks:
  - disease propagation on social contact graph
  - packet flow in cell phone communication
  - urban traffic and transportation
- Computational paradigms: Distributed computing

More Applications

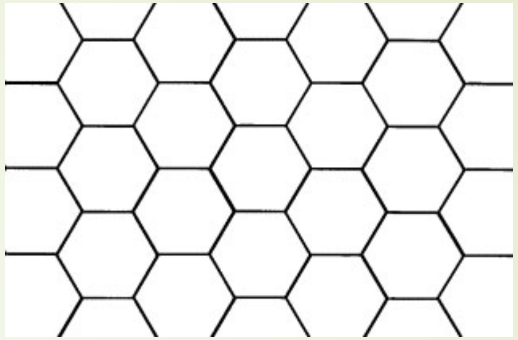


- Simulation of biological processes
- Simulation of cancer cells growth
- Image processing
- Pattern recognition
- Simulations of social movement
- Urban modeling
- Universal computers (embedded Turing machines)
- Cryptography
- …



"Cellular automata are sufficiently simple to allow detailed mathematical analysis, yet sufficiently complex to exhibit a wide variety of complicated phenomena.“ Stephen Wolfram (1983)

Homework



- Something other than a rectangular (cubic, etc.) grid
  - for example, if a plane is tiled with regular hexagons, those hexagons could be used as cells. In many cases the resulting cellular automata are equivalent to those with rectangular grids with specially designed neighborhoods and rules
- **Probabilistic cellular automata** - rules can be probabilistic rather than deterministic
- The neighborhood or rules could change over time or space
- **Continuous automata** - a CA extended so the valid states a cell can take continuous values, for example, the real number range [0,1]
  - such automata can be used to model certain physical reactions more closely
- More complex cell types –
  - remember history
  - with properties
- …

Demo

Is the universe a cellular automaton?