

COMPLEXITY: Exercise No. 4

Due: 27/11/02

1. (Test 95) Is the following problem NP-complete?

**1/2-INDEPENDENT SET:**

**Instance:** An undirected graph  $G = (V, E)$ .

**Question:** Does  $G$  have an independent set of size  $|V|/2$ ?

2. (Test 93) Is the following problem NP-complete?

**IS-CLIQUE:**

**Instance:** An undirected graph  $G$  and a positive integer number  $k$ .

**Question:** Does  $G$  contain a clique of size  $k$  or an independent set of size  $k$ ?

3. (Test 92) Is the following problem NP-complete?

**DOMINATING SET (DS):**

**Instance:** An undirected graph  $G = (V, E)$  and an integer  $k$ .

**Question:** Does  $G$  have a dominating set of size  $\leq k$ ? (a dominating set is a set  $U \subseteq V$ , such that for every  $v \in V \setminus U$  there is  $u \in U$  such that  $(u, v) \in E$ )

4. Is the following problem NP-complete?

**CONNECTED DOMINATING SET:**

**Instance:** An undirected graph  $G = (V, E)$  and a positive integer  $k$ .

**Question:** Does  $G$  contain a dominating set  $S$  with at most  $k$  vertices such that the subgraph of  $G$  induced by  $S$  (i.e., the graph  $G_S = (S, E \cap S \times S)$ ) is connected?

5. Is the following problem NP-complete?

**MINIMUM LEAF SPANNING TREE:**

**Instance:** An undirected graph  $G = (V, E)$  and a positive integer  $k$ .

**Question:** Is there a spanning tree for  $G$  in which the number of leaves is at most  $k$ ?

6. (Test 95) Is the following problem NP-complete?

**MAXIMUM LEAF SPANNING TREE:**

**Instance:** An undirected graph  $G$  and a positive integer  $k$ .

**Question:** Is there a spanning tree for  $G$  in which the number of leaves is at least  $k$ ?