

COMPLEXITY: Exercise No. 5

Due: 11/12/02

1. Show that MAX-CUT is **NP**-complete when the input graph is simple.
2. (Test 2000) Give a polynomial time algorithm that given a graph $G = (V, E)$ which contains an independent set of size $\frac{3}{4}|V|$ finds an independent set of size at least $\frac{1}{2}|V|$.
3. (Test 98) Consider the following problem:
MAX-3-CUT:
Instance: A simple undirected graph $G = (V, E)$ and a positive integer k .
Question: Is there a partition of V into 3 disjoint sets (V_1, V_2, V_3) such that the number of edges whose endpoints are in different sets is at least k ?
 - a. Prove that MAX-3-CUT is **NP**-complete.
 - b. Give a polynomial time approximation algorithm to the corresponding optimization problem (i.e. the problem of finding a 3-cut of maximum size). What is the approximation ratio?
4. (Test 98) Consider the TSP problem on a complete undirected graph with a length $l_{i,j} \geq 0$ for each edge (i, j) . Suppose that the lengths satisfy $l_{i,j} \leq l_{i,k} + 2l_{k,j}$ for all i, j, k .
Give an approximation algorithm for this problem.
What is the approximation ratio?
5. Consider the TSP problem on a complete undirected graph with a length $l_{i,j} \geq 0$ for each edge (i, j) . Suppose that the lengths satisfy $l_{i,j} \leq l_{i,k} + l_{k,j}$ for all i, j, k . Prove that this problem cannot be approximated to within a factor less than $1 + \frac{1}{n}$, whereas n is the number of vertices in the graph.