

**COMPLEXITY: Exercise No. 8**

**Due: 15/1/03**

1. Prove that if there is a  $PH$ -complete problem, then  $PH = \Sigma_k$  for some constant  $k$ .
2. Prove that if  $PH = PSPACE$  then  $PH = \Sigma_k$  for some constant  $k$ .
3. Prove that  $RP \subseteq NP$ .
4. Prove that  $NP \subseteq PP$ .
5. Consider the following alternative definition of  $ZPP$ :  
 $L \in ZPP$  iff there exists a probabilistic polynomial time TM  $M$  that answers TRUE, FALSE or QUIT, and:
  - If  $x \in L$  then  $M$  always returns TRUE or QUIT.
  - If  $x \notin L$  then  $M$  always returns FALSE or QUIT.
  - $\forall x \ Pr[M(x) = QUIT] \leq \frac{1}{2}$ .

Prove that this definition is equivalent to the definition we saw in class.

6. Prove that  $RP \cap co - RP \subseteq ZPP$ .