Exercise 1

Prove or disprove:

(1) $(5n)! = O(n!^5)$.
(2) *If* $f(n) = O(n)$ *then* $10^{f(n)} = O(2^n)$.
(3) *Every two functions* f, g *satisfy* $f = O(g)$ *or* $g = O(f)$.
(4) *There exists a function* f *such that* $f(n) = O(n^{1+\epsilon})$ *for any* $\epsilon > 0$ *but* $f(n) = \omega(n)$.

Exercise 2

Show the transition function (as in the recitation) of a one tape deterministic TM *that reverses a binary string. That is, given an input string* $x = x_1 x_2 \cdots x_n \in \{0,1\}^*$ *the output is* $\overleftarrow{x} = x_n x_{n-1} \cdots x_1$. *The machine is not allowed to use additional space on the tape beyond that occupied by the input.*

Exercise 3

For two languages L_1, L_2 *define* $L_1 \Delta L_2 = (L_1 \setminus L_2) \cup (L_2 \setminus L_1)$. *We say that a class* C *is closed under* Δ *if* $L_1, L_2 \in C$ *implies* $L_1 \Delta L_2 \in C$. *For each class decide if it is closed under* Δ *(or show that it is equivalent to an open question):* **P**, **NP**, **NP** $\cap$ **coNP**.

Exercise 4

For a language L *over alphabet* Σ *define*

$$L^{conc} = \{x \in \Sigma^* | \; \exists k \in \mathbb{N} \text{ and } \exists x_1, \ldots, x_k \in L \text{ such that } x = x_1 \ldots x_k\},$$
$$\overline{L} = \{x \in \Sigma^* \mid x \notin L\}.$$

Prove or disprove: $\overline{L^{conc}} = \overline{L}^{conc}$ *for any language* L.

GOOD LUCK