Exercise 1

In the $SAT \leq_p 3-SAT$ reduction we saw in class, we converted the formula $\varphi_{M,w}$ given by the Cook theorem to a CNF. $\varphi_{M,w}$ has a subformula that is in DNF form. The general conversion of $DNF \to CNF$ is exponential.

Prove that the conversion of $\varphi_{M,w}$ to CNF is polynomial in $|\varphi_{M,w}|$. (There is no need to prove the $CNF \to 3-CNF$ conversion).

Exercise 2

Prove that L is closed under $\leq_L$ reductions, that is: if $A \in L, A' \leq_L A$, then $A' \in L$. Be rigorous in your proof with respect to indicators and counters used, and what tape is data read/written from.

Exercise 3

Prove that the following problems are in NL:

(1) *Given an directed graph G, does G contain at most 2013 strongly connected components?*
(2) *Given an directed graph G, does G contain at least 2013 strongly connected components?*
(3) *Given an directed graph G, does G contain exactly 2013 strongly connected components?*
(4) *(bonus) Given an directed graph G and integer k, does G contain exactly k strongly connected components?*

Exercise 4

Prove that the following language is $NL-Complete$:

$$E_{DFA} = \{<M> \text{ s.t } M \text{ is a deterministic finite automaton, and } L(M) = \phi\}$$

Exercise 5

Assume that $EXP \neq NEXP$. Prove that $P \neq NP$.

GOOD LUCK