

Exercise 1

Prove that the following language is in NL :

$$\{\langle G \rangle \mid G \text{ is a Bipartite graph}\}$$

Exercise 2

Prove that the following language is in NL :

$$\{\langle G \rangle s, t \mid \text{exists exactly one path in } G \text{ from } s \text{ to } t\}$$

Exercise 3

Recall $Diam(G) = \max_{u,v \in V(G)} \{d(u, v)\}$. Is the following language in NL or in NPC ?

$$\{\langle G \rangle \mid G \text{ is a directed graph with } n \text{ vertices, } Diam(G) \leq \frac{n}{4}\}$$

Exercise 4

Prove that the following language is in $SPACE[\log(n) \cdot \log(\log(n))]$:

$$\{\langle G \rangle \mid G \text{ is a directed graph with } n \text{ vertices, } Diam(G) = \log(n)\}$$

Exercise 5

Let S be a set of families. Let X be the random variable s.t

$$\forall f \in S : X = \text{number of kids in } f, \text{ with probability } \frac{1}{|S|}$$

Furthermore, it is given that $\mathbb{E}[X] = 3$. What is the average number of siblings per child? If there is insufficient information, show a simple property (a number) of X that when given, allows you to answer.

GOOD LUCK