Exercise 1

Let A be a minimization problem on graphs, with solution a set of vertices.

Let A_α be the language: $A_\alpha = \{G = (V, E) \mid \frac{|A(G)|}{|V|} \leq \alpha\}$

(1) Prove that if there exists a poly-time (3)-approximation algorithm to A, then gap-$A[\frac{1}{6}, \frac{1}{2}]$ is poly-time solvable.

(2) Prove that if $A_\alpha \in P$ for all α then gap-$A[\frac{1}{6}, \frac{1}{2}]$ is poly-time solvable.

Exercise 2

IS - the optimization problem of finding $\frac{|maximal\ independent\ set|}{|V|}$

VC - the optimization problem of finding $\frac{|minimal\ vertex\ cover|}{|V|}$

Prove:

gap-IS$[\alpha, \beta]$ is poly-time solvable iff gap-VC$[1-\beta, 1-\alpha]$ is poly-time solvable.

Exercise 3

You saw in class a (2)-approximation algorithm for finding the minimal vertex cover in a graph. Show the approximation factor is tight. That is, show a graph on which this algorithm will return a set of vertices that is twice the size of the minimal vertex cover.

Exercise 4

Let C be the class of all languages that can be verified by a TM that has:

(a) Polynomial Work Tape

(b) Exponential Witness Tape that must be read bit by bit without going back

Prove: $C = PSPACE$

GOOD LUCK