Keep your answers as short and concise as possible, unnecessarily long answers answers will not be checked.

Exercise 1

Prove or disprove:

$gap{-}Diameter[log(n), \sqrt{n}]$ *can be solved using* $O(log(n) \cdot log(log(n)))$ *space.*

Exercise 2

(a) For every integer k*, construct a* $2CNF$ *formula with* $2k$ *clauses, such that every assignment satisfies exactly* k *clauses.*

(b) Let $MAX2SAT_{0.5+\epsilon} = \{\varphi \mid \varphi$ a $2{-}CNF$ formula, exists assignment satisfying $\geq (0.5+\epsilon)$ of φ's clauses$\}$

Prove that exists $\delta > 0$ *such that* $\forall \epsilon < \delta$, $MAX2SAT_{0.5+\epsilon}$ *is* $NP{-}Hard$

(Hint: Show a gap-preserving reduction from $gap{-}MAX2SAT[\frac{55}{80} + \epsilon', \frac{56}{80}]$*. You might want to use the construction from (a) in some way)*

Exercise 3

(a) Prove or disprove that $\forall \epsilon > 0$ *it is* $NP{-}Hard$ *to approximate* VC *to within factor* $\frac{17}{16} - \epsilon$.

(b) You saw in class that amplification was used to show that it is $NP{-}Hard$ *to approximate* IS *to within any constant factor. Does this method provide a better hardness of approximation result for* VC *than (a)?*

Exercise 4

$2{-}to{-}1$ CSG_E:

In a $2{-}to{-}1$ CSG_E *problem, for each* $e = (u, v) \in E$ *and coloring* $A(u) \in \Sigma$*, there are exactly two colorings of* v *that satisfy the constraint on* e*. (Where* Σ *is the set of possible colors.)*

Prove or disprove: For any $2{-}to{-}1$ CSG_E *problem* A*, gap-*$A\left[\frac{1}{|\Sigma|}, 1\right]$ *can be solved in constant time.*

Exercise 5

$2{-}to{-}1$ CSG_V:

In a $2{-}to{-}1$ CSG_V *problem, for each* $e = (u, v) \in E$ *and coloring* $A(u) \in \Sigma$*, there are exactly two colorings of* v *that satisfy the constraint on* e*. (Where* Σ *is the set of possible colors.)*

Prove or disprove: For any $2{-}to{-}1$ CSG_V *problem* A*, at least* $\frac{2}{|\Sigma|}$ *of the vertices can be colored.*

GOOD LUCK