Solution of A question from the TIRGUL

June 15, 2007

Question: You are given two sorted arrays, A and B, each of size n. Find their common median in time $O(\log n)$. You can assume that all elements in A and B are distinct.

Solution: Let $x = A[n/2], y = A[n/2]$. Suppose for a minute that $x < y$. Then the common median of A and B is equal to the common median of $A[n/2-1..n]$ and $B[1..n/2+1]$. Let's explain this: Firstly, we didn't drop the median, because there are less than $n/2$ elements smaller than x, and less than $n/2$ elements larger than y, and thus the median is at least x and at most y, while we have only dropped elements which are smaller than x or larger than y. Secondly, we have dropped the same number of elements which are larger than the median as those that are smaller than the median, so the median stays the same.

If $y < x$, then we simply switch the roles of x and y in the above explanation. Therefore, we have a recursive solution that always cuts n by roughly half, and thus the algorithm runs in time $O(\log n)$.