Data Structures - Assignment no. 1, October 25, 2007

Remarks:

- **Write both your name and your ID number very clearly on the top of the exercise. Write your exercises in pen, or in clearly visible pencil. Please write *very* clearly.**
- **Recall that 80% of the theoretical exercises must be submitted. The exercises can and must be worked on and submitted alone.**
- **Give correctness and complexity proofs for every algorithm you write.**
- **For every question where you are required to write pseudo-code, also explain your solution in words.**

1. Which of the following statements are true and which are false? Only give an answer, you do not have to explain:

   (a) $34n^{10} = o(2^n)$.
   (b) $4 \log n = O(8 \log n)$.
   (c) $n^3 + \log n = \Omega(n)$.
   (d) $5^n/50 = O(n)$.
   (e) $n + 1 = O(2^n)$.
   (f) $n + 1 = o(n)$.
   (g) $n/\log n = \Theta(\log n)$.

2. (a) Implement, in pseudo-code, a trenary (base-3) counter: You are given an infinite array such that each cell can only hold the digits 0,1,2, and you want the counter to support the operation *increment()* that increases the value of the counter by 1.
   (b) Suppose you start from a counter initialized to 0 and you perform m *increment*() operations. What is their total cost? What is the amortized time complexity of increment? Prove your answer in both the **bank** and the **potential** methods.

3. Let L be a singly-linked list that consists of n elements. Each element contains a pointer "next". Someone possibly made a mistake, and the final element of the list, instead of having a "null" pointer, might be pointing to an element inside the list. What does the following pseudo-code do? What is its worst-case time complexity?

   INPUT: x, which is a pointer to the first element of the list
   $y \leftarrow x.next$
   IF ($y == NULL$) THEN RETURN "true"

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WHILE (y ≠ x)
    y ← y.next
    IF (y == NULL) THEN RETURN "true"
    y ← y.next
    IF (y == NULL) THEN RETURN "true"
    x ← x.next
END WHILE
RETURN "false"
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