מבני נתונים

תרגול 3

ליאור שפירא

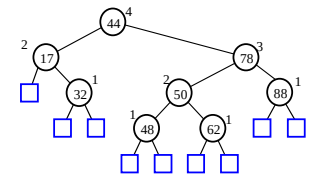
עצי חיפוש בינאריים מאוזנים

עצי חיפוש בינאריים

- ראינו כי עלות פעולה היא O(h)
- בהינתן עץ בעל n צמתים, גובהו המינימאלי יהיה O(logn)
- כיצד נגרום לעץ להיות מאוזן?

עצי AVL G.M. Adelson-Velskii / E.M.Landis 1962

- עץ חיפוש בינארי מאוזן (מבחינת גובה)
- עבור כל צומת פנימית v של עץ T
  - גובה תתי העצים מתחת ל-v יכול להיות שונה בעד 1



גובה של עץ AVL

- למה: גובה של עץ T המחזיק n מפתחות הוא O(logn)
- הוכחה:
  - נמצא את n(h), מס' מינימלי של צמתים פנימיים בעץ AVL בעל גובה h
  - בבירור n(1)=1, n(2)=2
  - עבור n≥3, עץ AVL בעל גובה h מכיל את השורש, תת עץ בגובה h-1 ותת עץ בגובה של לפחות h-2
  - ז"א: n(h)≥1+n(h-1)+n(h-2)

גובה של עץ AVL

$$n(h-1) > n(h-2) \Rightarrow n(h) \geq 2n(h-2)$$

$$n(h) \geq 2n(h-2)$$
$$n(h) \geq 4n(h-4)$$
$$\ldots$$
$$n(h) \geq 2^i n(h-2i)$$

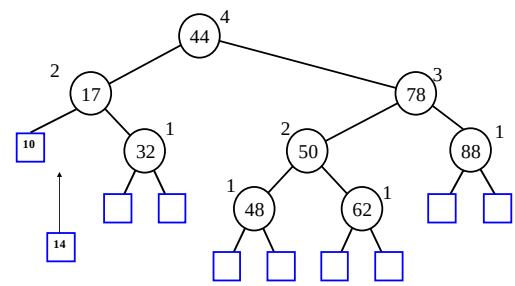
סדרה

$$\Rightarrow n(h) \geq 2^{h/2-1}$$ פתרון הסדרה

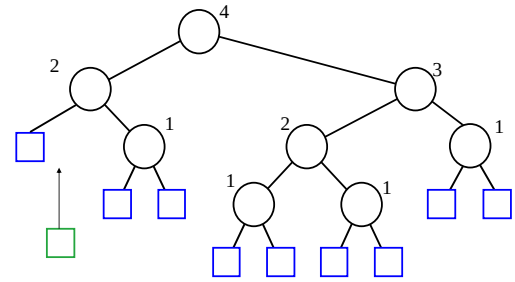
$$\Rightarrow h \leq 2\log n(h) + 2$$ לוג לשני הצדדים

$$\Rightarrow h = O(\log n)$$ מש"ל

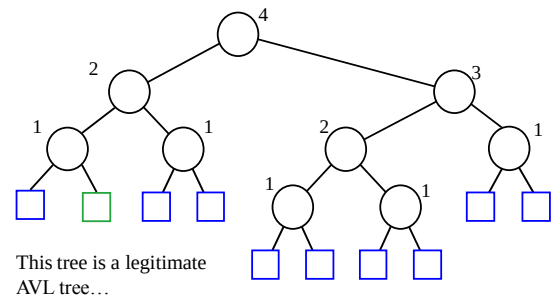
Insertion



נתעלם מהערכים, נתמקד באיזון העץ

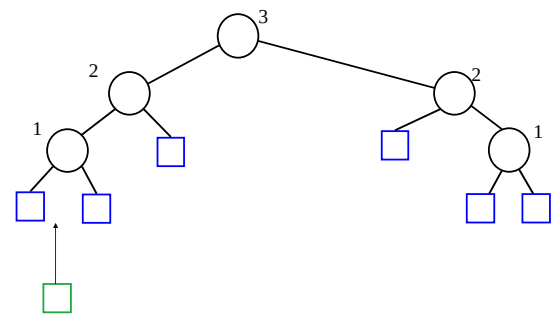


צומת פנימית חדשה

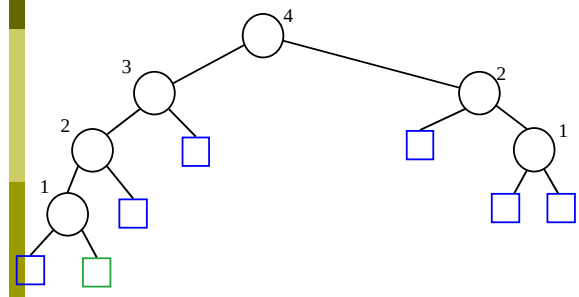


This tree is a legitimate AVL tree…

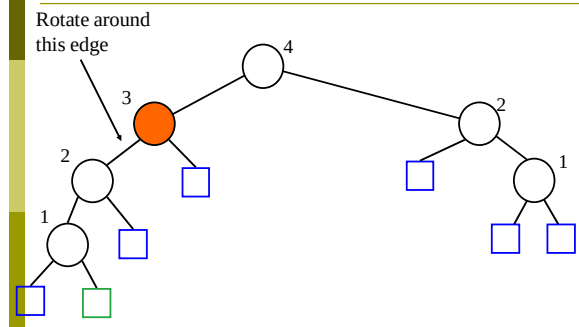
עוד דוגמה



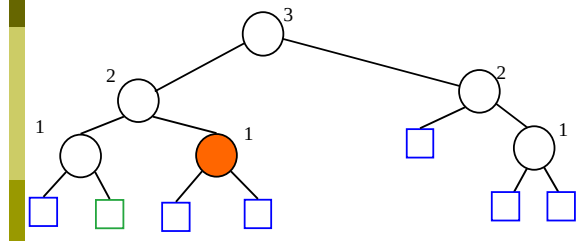
נוסיף את הצומת החדש



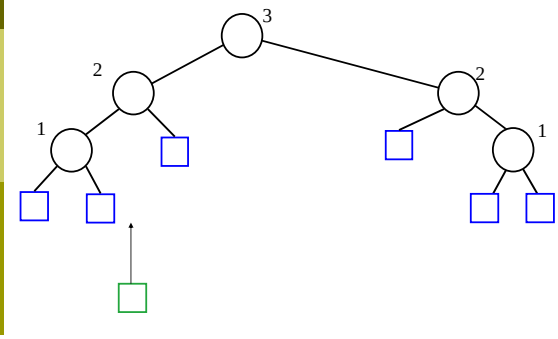
הפרנו את חוקיות העץ



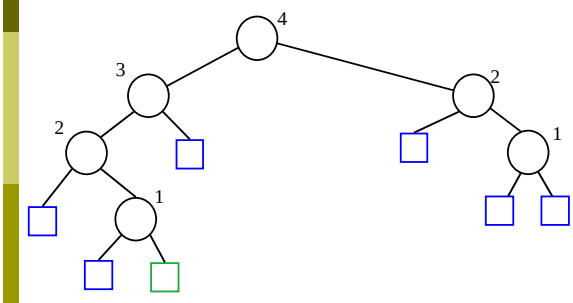
נתקן את השגיאות



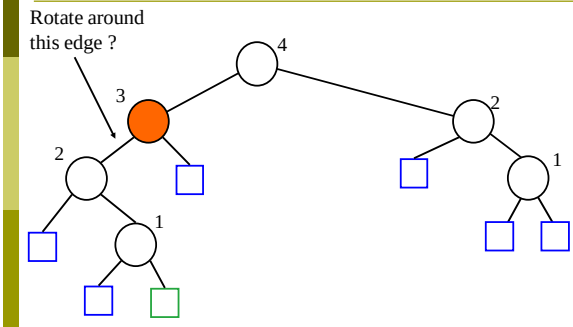
עוד דוגמה



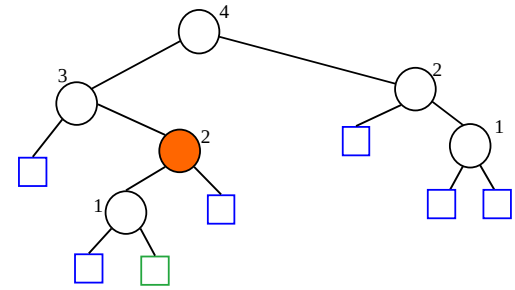
נוסיף את הצומת החדש



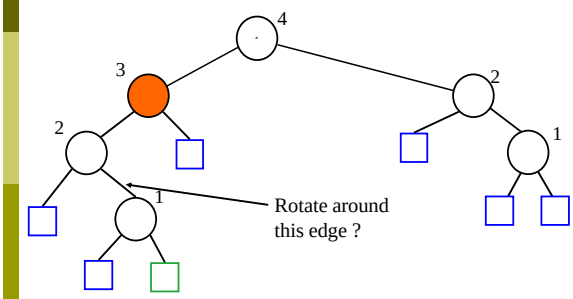
שוב קיבלנו הפרה של החוקים



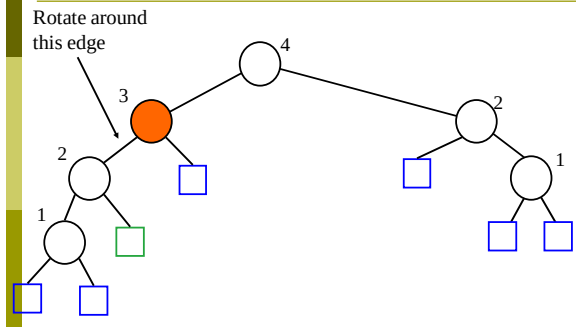
הפעם הרוטציה לא עוזרת



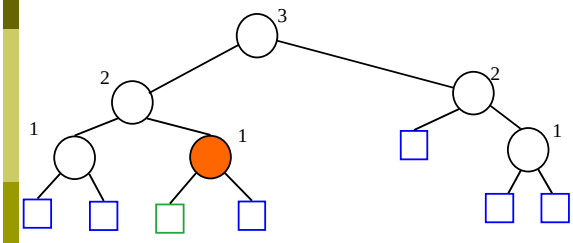
מה ניתן לעשות?



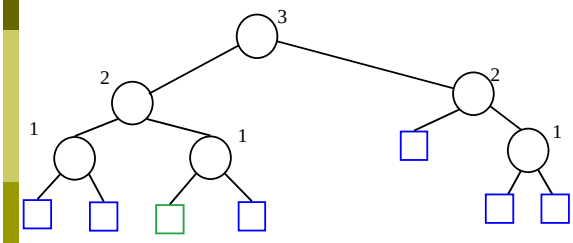
חזרנו למקרה הקודם



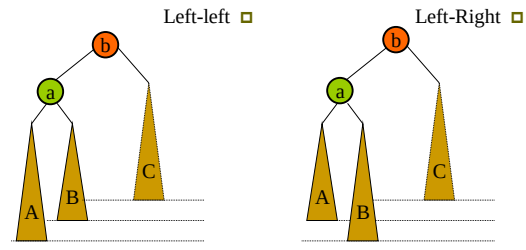
תיקנו את הפרת החוקים



אז מה האלגוריתם הכללי?

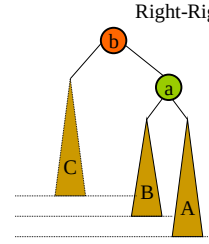


סוגי חוסר שיווי משקל

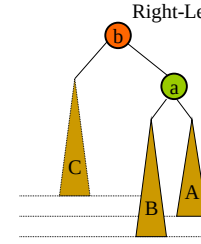


ותמונות המראה שלהם

Right-Right



Right-Left



נסתכל על הבעיה לוקאלית

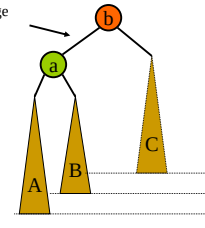
שני עקרונות מנחים

- Imbalance will only occur on the path from the inserted node to the root (only these nodes have had their subtrees altered - local problem)
- Rebalancing should occur at the *deepest unbalanced node* (local solution too)

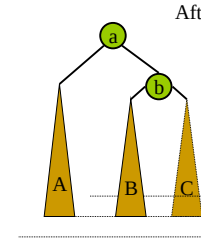
Left-left ➔ single rotation

Rotate around this edge

Before

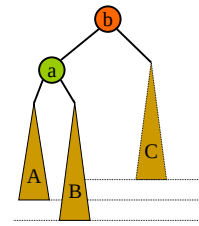


After



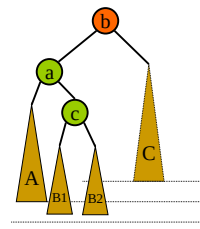
Left-Right fixing

• Before:



Lets look at this more carefully

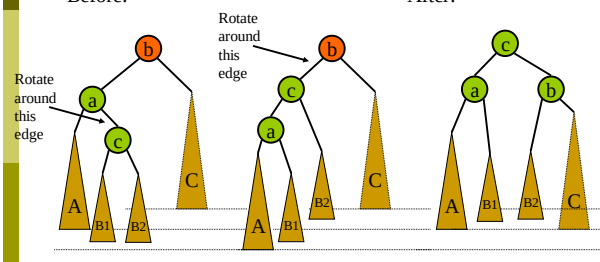
• Before:



We need two rotations here…(double rotation)

• Before:

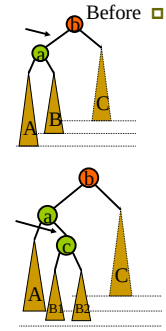
• After:



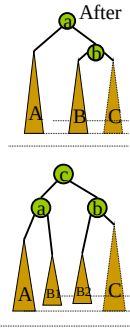
בהוספת צומת חוסר האיזון "מוגבל"

- שימו לב שבשני המקרים של חוסר איזון בהוספת צומת, לאחר הrebalancing, גובה תת העץ שהשורש שלו הוא הצומת שהייתה בחוסר איזון, נשאר אותו דבר! ז"א שלאחר איזון (רוטציה או שתיים) אין צורך להמשיך ולבדוק חוסר איזון יותר גבוה בעץ, כי בוודאות אין כזה!

- Before



- After



סיכום

- We fix the first node x on the way up where there is a violation
- After the fix, x is at the same height as it was before, so no nodes further up towards the root will need to be updated.
- Can implement using just 2 bits per node for rebalancing (the difference between the heights of the children

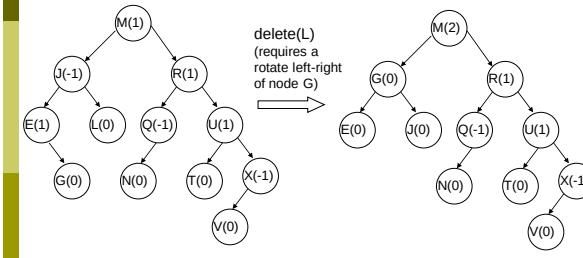
AVL Tree Delete

- An AVL delete is similar to a regular binary tree delete
  - search for the node
  - remove it
    - zero children: replace it with null
    - one child: replace it with the only child
    - two children: replace it with right-most node in the left subtree

AVL Tree Delete

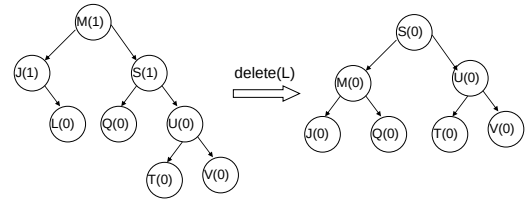
- Complications arise from the fact that deleting a node can unbalance a number of its ancestors
  - insert only required you find the first unbalanced node
  - delete will require you to go all the way back to the root looking for imbalances
    - Must balance any node with a ±2 balance factor (+2 the left sub-tree is 2 levels deeper, -2 the right sub-tree is 2 levels deeper)

AVL Tree Delete



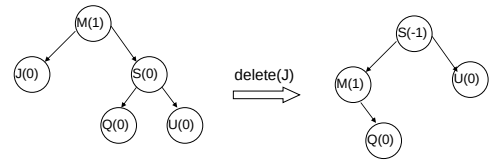
Notice that even after fixing J, M is still out of balance

Rotating Nodes



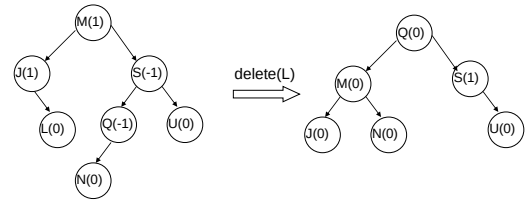
Deleting a node in the left sub-tree (M's balance becomes 2).
Need to rotate M with it's right sub-tree.
S's balance factor is 1 before rotate.
Just do a rotate left of node S.
Notice that the height of the tree *does* change in this case.

Rotating Nodes



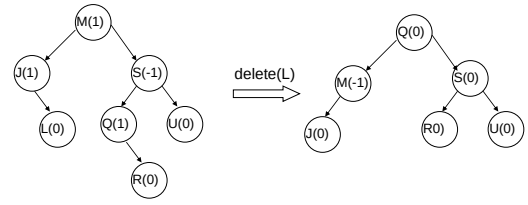
Deleting a node in the left sub-tree (M's balance becomes 2).
Need to rotate M with it's right sub-tree.
S's balance factor is 0 before rotate.
Just do a rotate left of node S.
Notice that the height of the tree *does not* change in this case.

Rotating Nodes



Deleting a node in the left sub-tree (M's balance becomes 2).
Need to rotate M with it's right sub-tree.
S's balance factor is -1 before rotate.
Need to do a right rotate of Q with S and then a left rotate of Q with M.
Notice that the height of the tree changes.

Rotating Nodes



Deleting a node in the left sub-tree (M's balance becomes 2).
Need to rotate M with it's right sub-tree.
S's balance factor is -1 before rotate.
Need to do a right rotate of Q with S and then a left rotate of Q with M.
Notice that the height of the tree changes.

זמני ריצה

- a single restructure is O(1)
  - using a linked-structure binary tree
- find is O(log n)
  - height of tree is O(log n), no restructures needed
- insert is O(log n)
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)
- remove is O(log n)
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)

לינקים מעניינים

- AVL Trees on Wikipedia
- AVL Tree applet
- Nice Red-Black tree demo
- Another AVL/RB tree applet

The End