קורס גרפיקה ממוחשבת

2008 סמסטר ב'

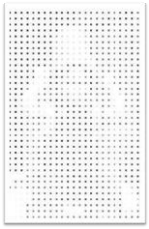
Image Processing

חלק מהשקפים מעובדים משקפים של פרדו דוראנד, טומס פנקהאוסר ודניאל כהן-אור 1

What is an image?

- An image is a discrete array of samples representing a continuous 2D function

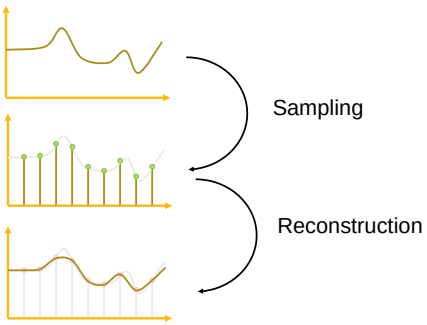
Continuous function



Discrete samples

2

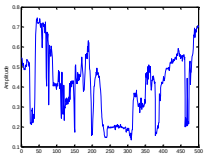
Sampling and Reconstruction



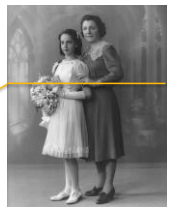
Converting to digital form

- Convert continuous sensed data into digital form

Quantization



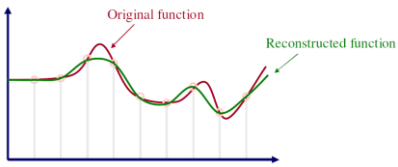
Sampling



3

Sampling Theory

- How many samples are required to represent a given signal without loss of information?
- What signals can be reconstructed without loss for a given sampling rate?



Sampling and Reconstruction

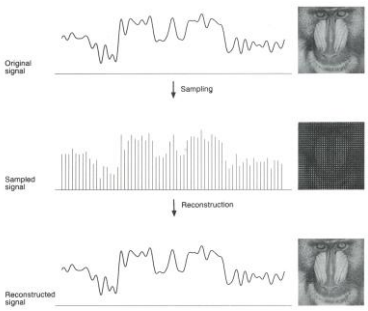


Figure 19.9 FvDFH

Spectral Analysis

- So our image (function f(x,y)) describes how the signal changes over "time" (x and y axes)
- Aliasing occurs when we use too few samples (what is enough?)
- The more an image changes, the more we need to sample it.
- How do we measure how fast a signal changes?
  - Frequencies

8

Aliasing

- What happens when we use too few samples?
  - Aliasing

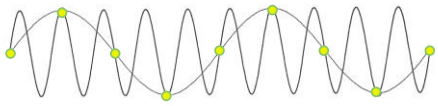


Figure 14.17 FvDFH

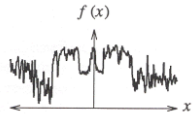
Fourier

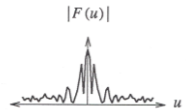
- Joseph Fourier discovered in 1822 that
  - Any periodic function can be expressed as the sum of sines and/or cosines if different frequencies (*Fourier Series*)
  - Even functions that are not periodic can be expressed as the integral of sines and/or cosines (*Fourier Transform*)
  - Initial application was in heat diffusion

10

Spectral Analysis

- Spatial domain:
  - Function: f(x)
  - Filtering: convolution
- Frequency domain:
  - Function: F(u)
  - Filtering: multiplication





Any signal can be written as a sum of periodic functions.

Fourier Transform (1D)

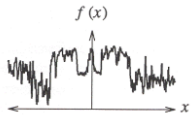
- Fourier transform:

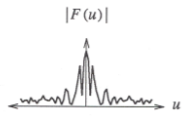
$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xu} dx$$

- Inverse Fourier transform:

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{+i2\pi ux} du$$

Fourier Transform (1D)





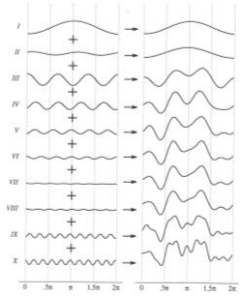


Figure 2.6 Wolberg

Image Processing

- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
  - Sharpen
  - Emboss
  - Median
- Quantization
  - Uniform Quantization
  - Floyd-Steinberg dither
- Warping
  - Scale
  - Rotate
  - Warps
- Combining
  - Composite
  - Morph

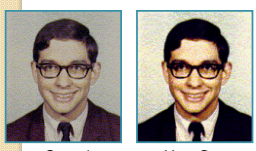
Sampling Theorem

- A signal can be reconstructed from its samples, if the original signal has no frequencies above 1/2 the sampling frequency - Shannon
- The minimum sampling rate for bandlimited function is called "Nyquist rate"

A signal is bandlimited if its highest frequency is bounded. The frequency is called the bandwidth.

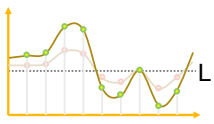
Adjusting Contrast

- Compute mean luminance L for all pixels
  - luminance = 0.30*r + 0.59*g + 0.11*b
- Scale deviation from L for each pixel component
  - Must clamp to range (e.g., 0 to 1)



Original

More Contrast

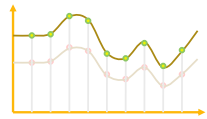


Adjusting Brightness

- Simply scale pixel components
  - Must clamp to range (e.g., 0 to 1)

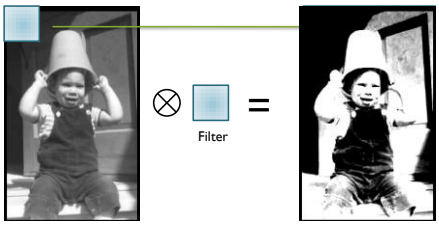
Original

Brighter



Linear Filtering (Spatial Domain)

- Convolution
  - Each output pixel is a linear combination of input pixels in neighborhood with weights prescribed by a filter



18

Image Processing

- Pixel operations
  - Add random noise
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- Combining
  - Composite
  - Morph

More on blur (lowpass filters)

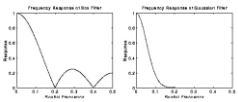
- We can either take a uniform kernel (mean filter)

$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

- Or a Gaussian kernel

$$\begin{bmatrix} 1/16 & 2/16 & 1/16 \\ 2/16 & 4/16 & 2/16 \\ 1/16 & 2/16 & 1/16 \end{bmatrix}$$

- A Gaussian kernel tends to provide gentler smoothing and preserve edges better



20

Adjust Blurriness

- Convolve with a filter whose entries sum to one
  - Each pixel becomes a weighted average of its neighbors


Original


Blur

Filter = $\begin{bmatrix} 1/16 & 2/16 & 1/16 \\ 2/16 & 4/16 & 2/16 \\ 1/16 & 2/16 & 1/16 \end{bmatrix}$

What do you think happens in the frequency domain?

Sharpen

- Sum detected edges with original image


Original


Sharpened

Filter = $\begin{bmatrix} -1 & -1 & -1 \\ -1 & +9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

22

Edge Detection

- Convolve with a filter that finds differences between neighbor pixels


Original


Detect edges

Filter = $\begin{bmatrix} -1 & -1 & -1 \\ -1 & +8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

Non-linear filtering

- Any operation on a neighborhood around each pixel
- For example: Selecting the **median** value of the neighborhood

24

Emboss

- Convolve with a filter that highlights gradients in particular directions


Original


Embossed

Filter = $\begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

23

Quantization

- Reduce intensity resolution
  - Frame buffers have limited number of bits per pixel
  - Physical devices have limited dynamic range

26

Image Processing

- Pixel operations
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Uniform Quantization

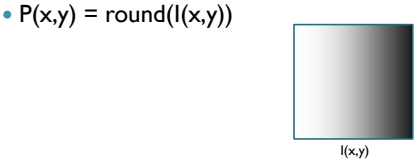
- Images with decreasing bits per pixel:

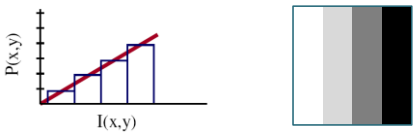
8 bits 4 bits 2 bits 1 bit

28

Uniform Quantization

- P(x,y) = round(I(x,y))





P(x,y) – 2 bits per pixel

27

Dithering

- Distribute errors among pixels
  - Exploit spatial integration in our eye
  - Display greater range of perceptible intensities

Original (8 bits) | Uniform Quantization (1 bit) | Floyd-Steinberg Dither (1 bit)

Reducing effects of Quantization

- Dithering
  - Random dither
  - Ordered dither
  - Error diffusion dither
- Halftoning
  - Classical halftoning

29

Random Dither

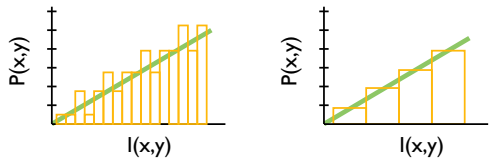

Original
(8 bits)


Uniform
Quantization
(1 bit)


Random
Dither
(1 bit)

Random Dither

- Randomize quantization errors
  - Errors appear as noise



$$P(x, y) = \text{trunc}(I(x, y) + \text{noise}(x,y) + 0.5)$$

1 bit

Ordered Dither

- Bayer's ordered dither matrices

$$D_n = \begin{bmatrix} 4D_{n/2} + D_2(1,1)U_{n/2} & 4D_{n/2} + D_2(1,2)U_{n/2} \\ 4D_{n/2} + D_2(2,1)U_{n/2} & 4D_{n/2} + D_2(2,2)U_{n/2} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \qquad D_4 = \begin{bmatrix} 15 & 7 & 13 & 5 \\ 3 & 11 & 1 & 9 \\ 12 & 4 & 14 & 6 \\ 0 & 8 & 2 & 10 \end{bmatrix}$$

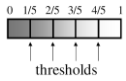
Basic idea: organize successive integers such that the average distance between two successive numbers in the map is as large as possible

Ordered Dither

- Pseudo-random quantization errors
  - Matrix stores pattern of threshholds

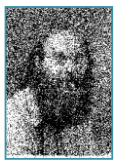
$$D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

For each pixel (x,y)
oldpixel = I(x,y) +D(x mod n,y mod n)
P(x,y)= find_closest_color(oldpixel)



Ordered Dither

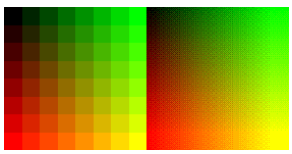

Original
(8 bits)


Random
Dither
(1 bit)


Ordered
Dither
(1 bit)

Ordered Dither

- An example
  - Palette consists of 8 red tones, 8 green tones and their combinations (64 colors)
  - Original image had 19600 colors


Undithered Dithered

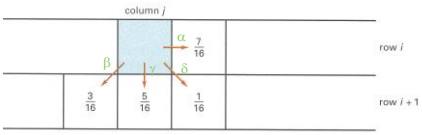
35

Floyd-Steinberg Algorithm

```
for (x = 0; x < width; x++) {
        for (y = 0; y < height; y++) {
                P(x,y) = trunc(I(x,y) + 0.5)
                e = I(x,y) - P(x,y)
                I(x,y+1) += α*e;
                I(x+1,y-1) += β*e;
                I(x+1,y) += γ*e;
                I(x+1,y+1) += δ *e;
        }
}
```

Error Diffusion Dither

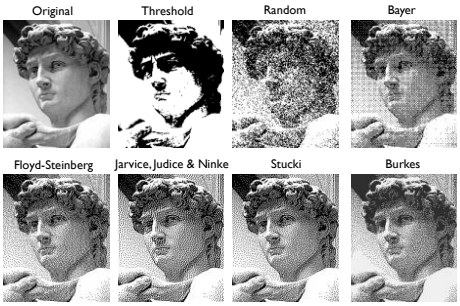
- Spread quantization error over neighbor pixels



$$\alpha + \beta + \gamma + \delta = 1.0$$

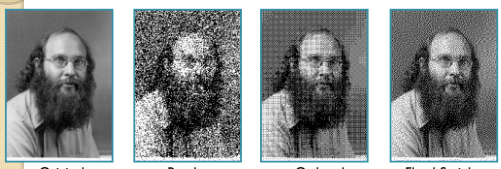
Figure 14.42 from H&B

More examples



40

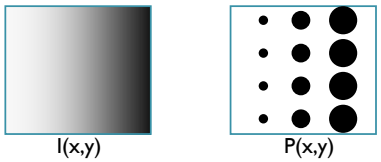
Error Diffusion Dither



Original (8 bits) | Random Dither (1 bit) | Ordered Dither (1 bit) | Floyd-Steinberg Dither (1 bit)

Classical Halftoning

- Use dots of varying size to represent intensities
  - Area of dots proportional to intensity in image



I(x,y) P(x,y)

Reducing effects of Quantization

- Dithering
  - Random dither
  - Ordered dither
  - Error diffusion dither
- Halftoning
  - Classical halftoning

41

Halftone patterns

- Use cluster of pixels to represent intensity
  - Trade spatial resolution for intensity resolution

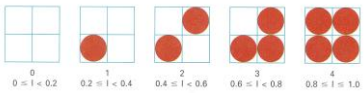
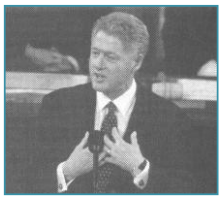
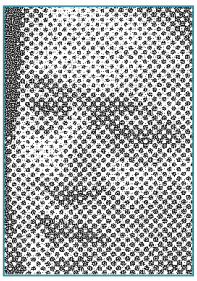


Figure 14.37 from H&B

Classical Halftoning



Newspaper Image



From New York Times, 9/21/99

Image Processing

- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
  - Sharpen
  - Emboss
  - Median
- Quantization
  - Uniform Quantization
  - Floyd-Steinberg dither
- Warping
  - Scale
  - Rotate
  - Warps
- Combining
  - Composite
  - Morph

Halftone patterns

- How many intensities in a n x n cluster?

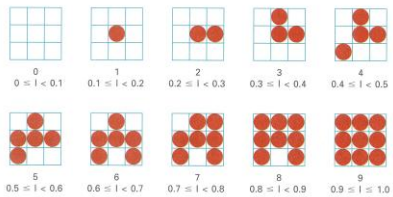
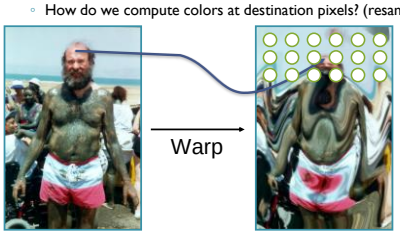


Figure 14.37 from H&B

Image Warping

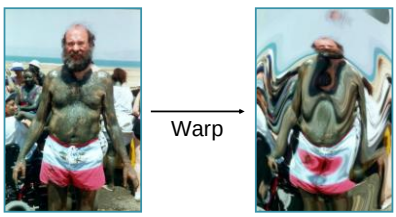
- Issues
  - How do we specify where every pixel goes? (mapping)
  - How do we compute colors at destination pixels? (resampling)



Source image Destination image

Image Warping

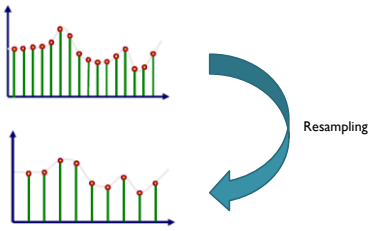
- Move pixels of image



Source image Destination image

Image Warping

- Image warping requires resampling of image



50

Example

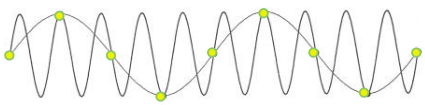
- Image Scaling
  - (x',y') = (sx*x, sy*y);
  - I(x',y') = ?



49

Aliasing (again)

- In general:
  - Artifacts due to under-sampling or poor reconstruction
- Specifically, in graphics:
  - Spatial aliasing
  - Temporal aliasing



Under-sampling

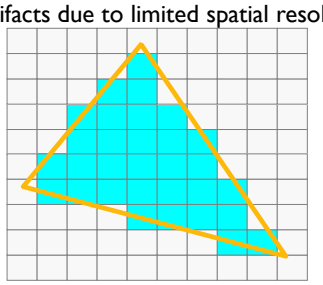
Figure 14.17 FvDFH

BACK TO SAMPLING

51

Spatial Aliasing

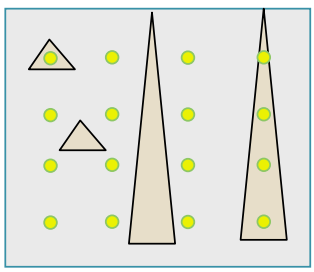
- Artifacts due to limited spatial resolution



"Jaggies"

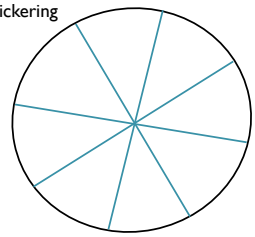
Spatial Aliasing

- Artifacts due to limited spatial resolution



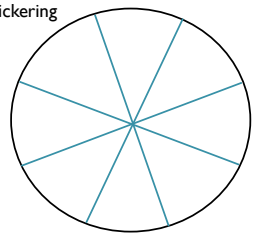
Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering



Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering



Temporal Aliasing

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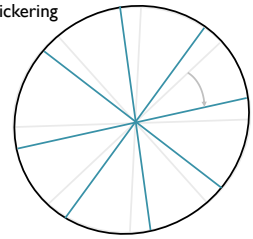

Temporal Aliasing

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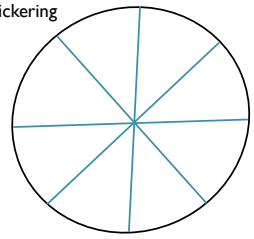
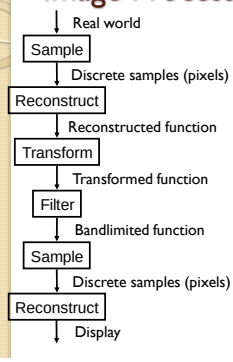


Image Processing

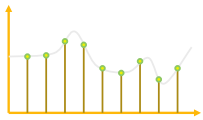


Antialiasing

- Sample at higher rate
  - Not always possible
  - Doesn't always solve problem
- Pre-filter to form bandlimited signal
  - Form bandlimited function (low-pass filter)
  - Trades aliasing for blurring

Image Processing

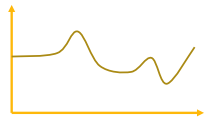
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Discrete Samples

Image Processing

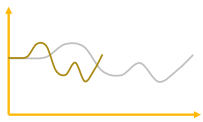
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Continuous Function

Image Processing

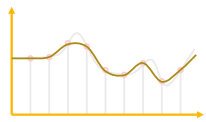
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Transformed Function

Image Processing

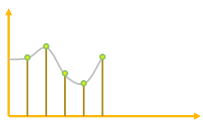
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Reconstructed Function

Image Processing

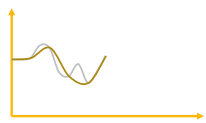
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



Discrete samples

Image Processing

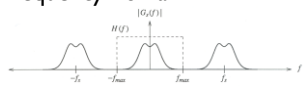
Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display



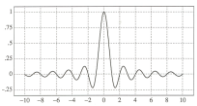
Bandlimited Function

Ideal Bandlimiting Filter

- Frequency domain



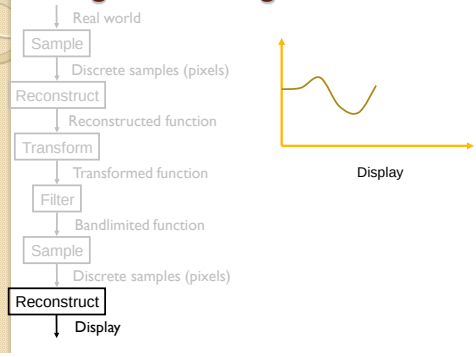
- Spatial domain



$$Sinc(x) = \frac{\sin \pi x}{\pi x}$$

Figure 4.5 Wolberg

Image Processing



Triangle Filter

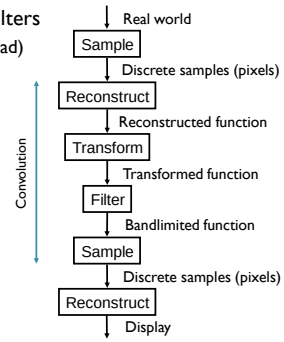
- Convolution with triangle filter

Input Output

Figure 2.4 Wolberg

Practical Image Processing

- Finite low-pass filters
  - Point sampling (bad)
  - Triangle filter
  - Gaussian filter



AND BACK TO WARPING

Gaussian Filter

- Convolution with Gaussian filter

Input Output

Figure 2.4 Wolberg

Image Resampling

- Compute weighted sum of pixel neighborhood
  - Output is weighted average

```
dst(u,v)=0;
for(ix=u-w;ix<=u+w;ix++)
  for(iy=v-w;iy<=v+w;iy++)
    d=dist between (ix,iy) and (u,v)
    dst(u,v) += k(ix,iy) * src(ix,iy)
```

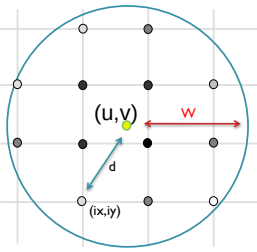


Image Resampling

- What if we are resampling a 2D image?

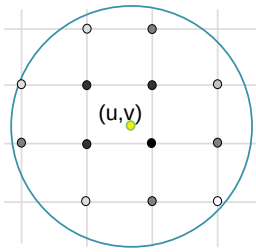
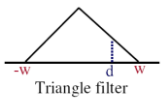


Image Resampling

- For isotropic Triangle and Gaussian filters, k(ix,iy) is a function of d and w



Triangle filter

k(i,j)=1 - d/w

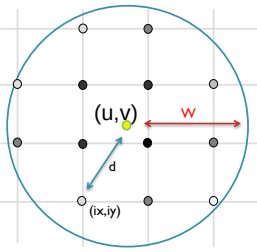
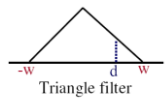


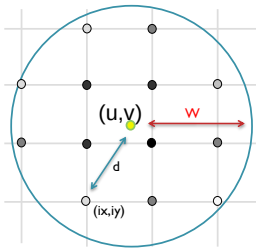
Image Resampling

- For isotropic Triangle and Gaussian filters, k(ix,iy) is a function of d and w



Triangle filter

k(i,j)=1 - d/w

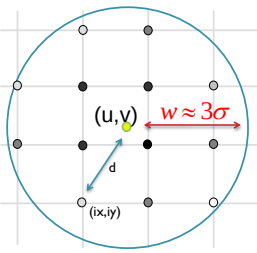


Gaussian Filtering

- Kernel is a Guassian function

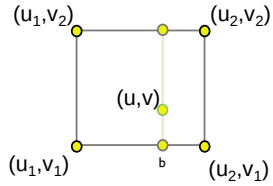
Gaussian Function

$$G_\sigma(d) = 2^{-(d/\sigma)^2}$$



Triangle Filtering (width <= 1)

- Bilinearly interpolate four closest pixels
  - a = linear interpolation of src(u_1,v_2) and src(u_2,v_2)
  - b = linear interpolation of src(u_1,v_1) and src(u_2,v_1)
  - dst(x,y) = linear interpolation of "a" and "b"

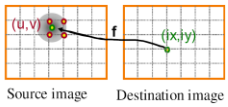


How do we resample?

- Point sampling
  - Simple but causes aliasing

```
Float resample(src,u,v,w) {
  int iu = round(u);
  int iv = round(v);
  return src(iu,iv);
}
```

- Triangle and Gaussian
  - Algorithm as we saw earlier



80

Image Scale

- Scale (src, dst, sx, sy):

```
w ≅ max(1/sx,1/sy);
for (int ix = 0; ix < xmax; ix++) {
  for (int iy = 0; iy < ymax; iy++) {
    float u = ix / sx;
    float v = iy / sy;
    dst(ix,iy) = resample(src,u,v,k,w);
  }
}
```

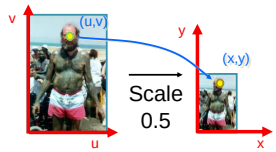
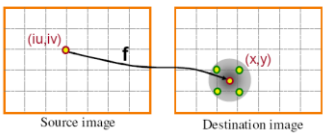


Image Warping (in General)

- Alternative (forward)

```
Warp(src, dst) {
  for (int iu = 0; iu < umax; iu++) {
    for (int iv = 0; iv < vmax; iv++) {
      float x = f_x(iu,iv);
      float y = f_y(iu,iv);
      float w ≈ 1 / scale(x, y);
      Splat(src(iu,iv), x, y, w);      weighting ???
    }
  }
}
```

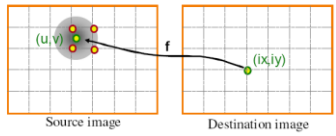


82

Image Warping (in General)

- Reverse Mapping

```
Warp(src, dst) {
  for (int ix = 0; ix < xmax; ix++) {
    for (int iy = 0; iy < ymax; iy++) {
      float w ≈ 1 / scale(ix, iy);
      float u = f_x^-1(ix,iy);
      float v = f_y^-1(ix,iy);
      dst(ix,iy) = Resample(src,u,v,w);
    }
  }
}
```



81

That's it for today

- Next time?
  - Finishing corners on image processing
  - Transformations and Projections
  - Rendering

83