Poisson Image Editing

Exercise 1

Due date: 30.03.09

General Description

The purpose of this exercise is to understand and implement a Poisson seamless cloning image editing tool.

Part 1: Smooth image completion

Part 2: Poisson seamless cloning

Input Images

source image

target image

Simple Cloning Result

Poisson Seamless Cloning Result

Some More Results

source images

target image

Some More Results

source images

simple cloning

Some More Results

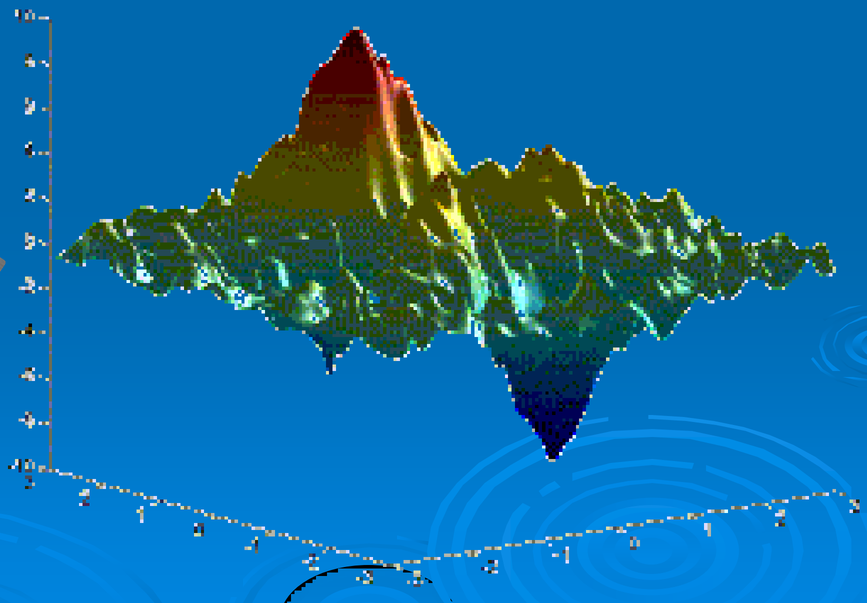
source images

Poisson seamless cloning

Part 1

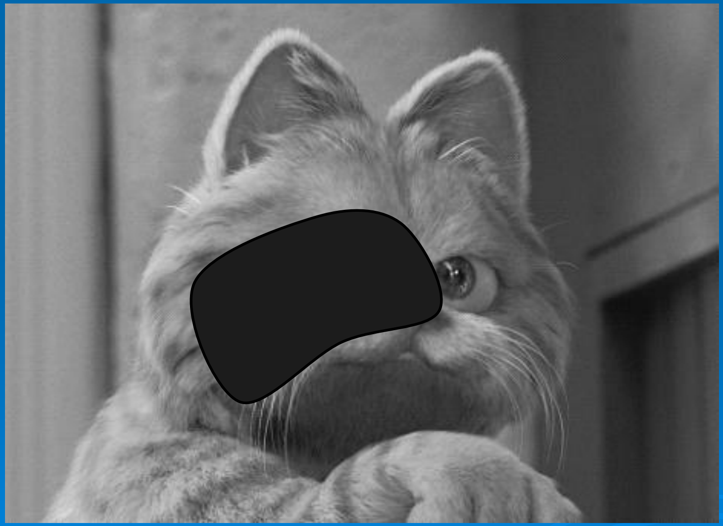
Smooth Completion

Image as a 2D Function



Smooth Image Completion

What if there is a missing area ?



f* – the known image
Scalar 2D function from (x,y) to grayscale value.

f - the image in the unknown area

Ω – the unknown area (domain of f)

$$\arg\min_{f} \iint_{\Omega} |\nabla f|^2 \quad s.t. \quad f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

Will complete the area as smoothly as possible.

Smooth Image Completion (Cont.)

Finding the minimum:

$$\arg\min_{f} \iint_{\Omega} |\nabla f|^2 \quad s.t. \; f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

Euler-Lagrange

$$\Delta f = 0 \; over \, \Omega \quad s.t. \; f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Discrete Aprx:

$$\frac{\partial f}{\partial x} \cong f_{x+1,y} - f_{x,y}$$

$$\frac{\partial^2 f}{\partial x^2} \cong f_{x+1,y} - 2 f_{x,y} + f_{x-1,y}$$

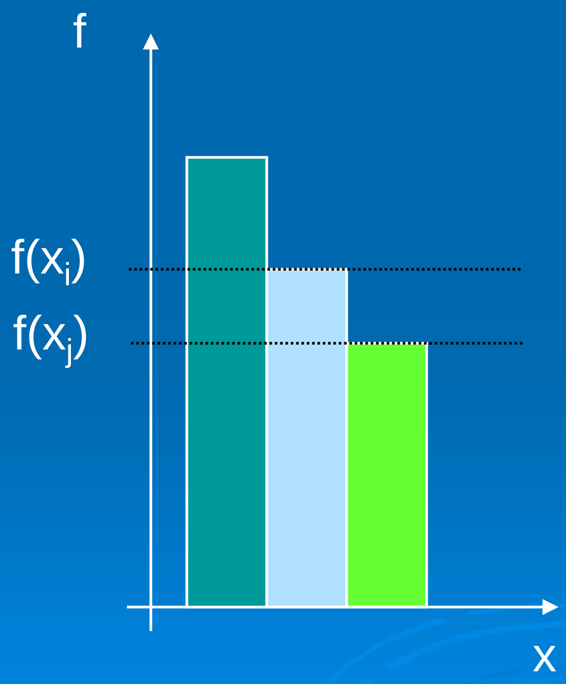
$$\Delta f(x,y) \cong f_{x+1,y} - 2 f_{x,y} + f_{x-1,y} +$$

$$f_{x,y+1} - 2 f_{x,y} + f_{x,y-1} =$$

$$= f_{x+1,y} + f_{x-1,y} + f_{x,y+1} + + f_{x,y-1} - 4 f_{x,y} = 0$$

Discrete Derivate in 1D

- Given a discrete function $f(x_i)=f_i$



$$\frac{df}{dx}(x_i) \cong \frac{f_{i+1} - f_i}{x_{i+1} - x_i}$$

Smooth Image Completion (Solving)

- Each $f_{x,y}$ is an unknown variable x_i, total of N variables (covering the unknown pixels)

$$f_{x,y-1}+f_{x-1,y}-4f_{x,y}+f_{x+1,y}+f_{x,y+1}=0 \quad \Rightarrow \quad x_{i-w}+x_{i-1}-4x_i+x_{i+1}+x_{i+w}=0$$

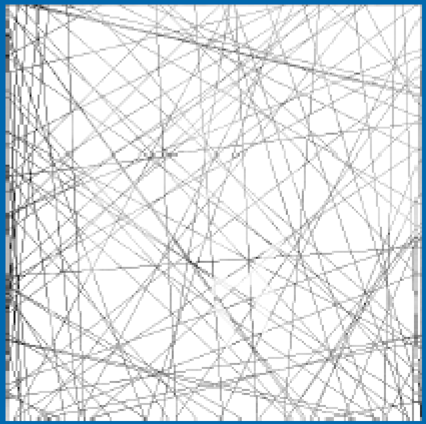
- Reduces to the sparse algebraic system:

$$\underbrace{\begin{pmatrix} 1 & & & 1 & -4 & 1 & & 1 & & \\ & 1 & & & 1 & -4 & 1 & & 1 & \\ & & 1 & & & 1 & -4 & 1 & & 1 \\ & & & & & & & & & \end{pmatrix}}_{N \times N} \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ \\ x_N \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ b_1 \\ b_2 \\ 0 \\ 0 \\ 0 \\ b_3 \\ 0 \end{pmatrix}$$

Known values of f() contribute to the left side

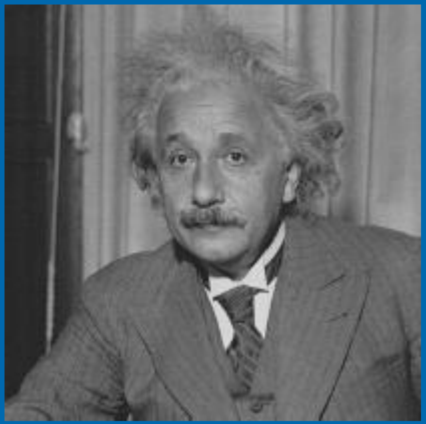
$$x_{i-w}+x_{i-1}-4x_i+x_{i+1}=-f(x,y+1)$$

Example



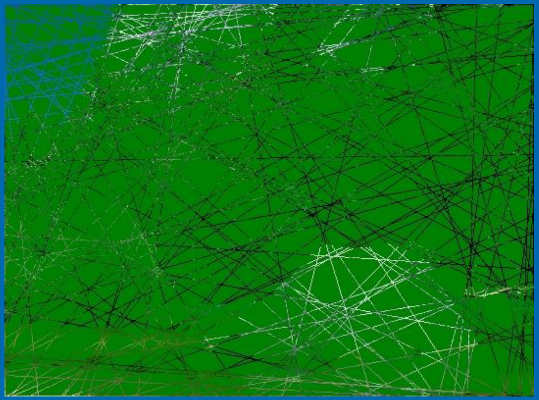
input

result



ground truth

Another Example



input

result

ground truth

Part 2

Poisson Cloning

Poisson Cloning: "Guiding" the completion

- We can guide the completion from part1 to fill the hole using gradients from another source image
- Reverse: Seek a function f whose gradients are closest to the gradients of the source image

Poisson Cloning

$$\arg\min_{f} \iint_{\Omega} |\nabla f - G|^2 \quad s.t. \; f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

$$G = \nabla source\ image$$

(forward difference)

$$\Delta f = div\, G \; over\, \Omega \quad s.t. \; f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

$$div\, G = \frac{\partial G}{dx} + \frac{\partial G}{dy} \cong G_x(x,y) - G_x(x-1,y) + G_y(x,y) - G_y(x,y-1)$$

(backward difference)

Poisson Cloning (Solving)

- Each $f_{x,y}$ is a variable x_i as before, solving

  $f_{x,y-1}+f_{x-1,y}-4f_{x,y}+f_{x+1,y}+f_{x,y+1}=divG(x,y)$
  $\Rightarrow x_{i-w}+x_{i-1}-4x_i+x_{i+1}+x_{i+w}=divG(x,y)$

- As before this reduces to a sparse algebraic system