Animation

ליאור שפירא

גרפיקה ממוחשבת

סמסטר א' תש"ע

מבוסס על שקפים של תומס פנקהאוסר, פרינסטון

הצצה לשבוע הבא...

- רן גל יספר לנו על iWires

- טל, מהנדס מאלביט יספר לנו על פרויקטים גרפיים מעניינים שהם מפתחים

Today

- Animation
- Simulation
  - Dynamics
  - Kinematics
- Deformations

Computer Animation

- What is animation?
  - Make objects change over time according to scripted actions

Pixar

- What is simulation?
  - Predict how objects change over time according to physical laws

University of Illinois

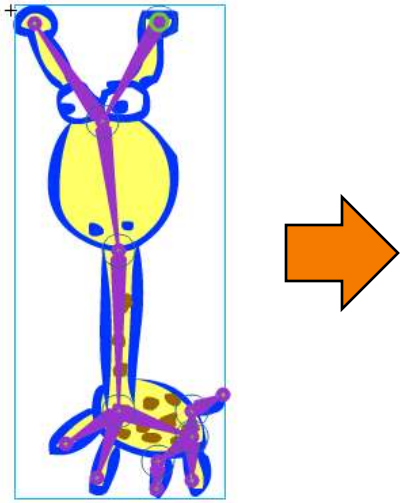
Computer Animation

Pixar Parody

Pixar

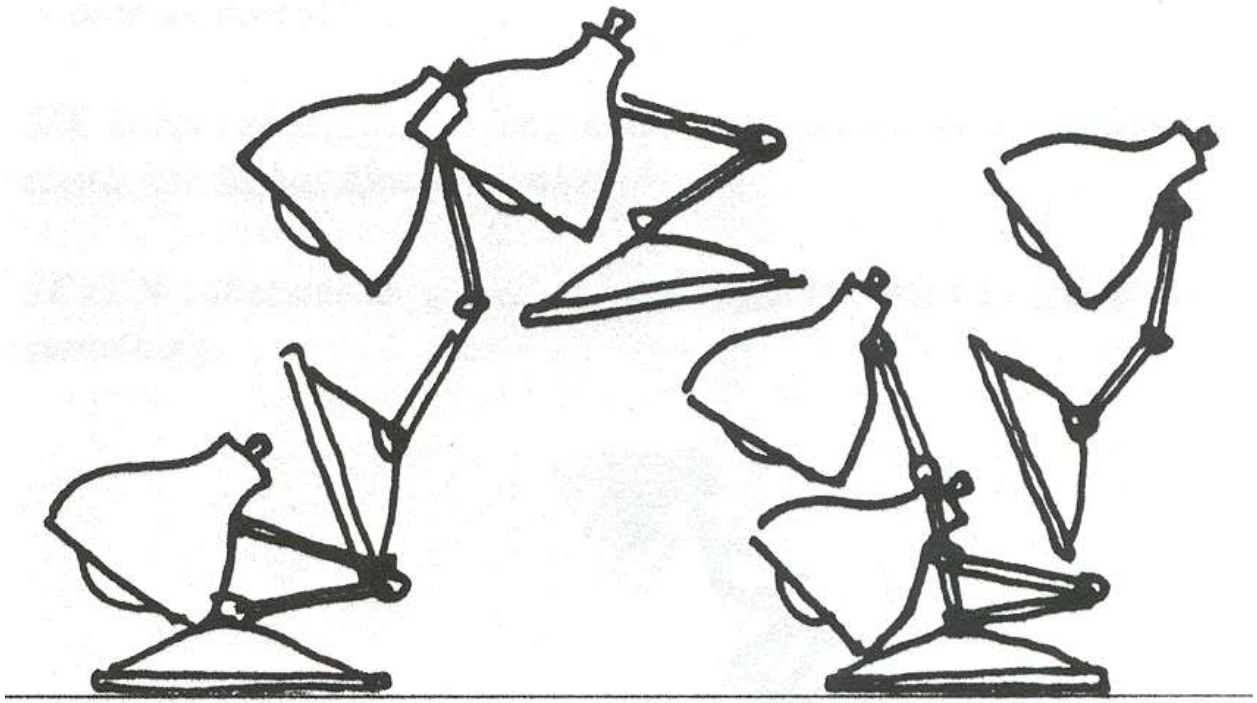
Outline

- Keyframe animation
- Adding inverse kinematics
- Adding dynamics



Keyframe Animation

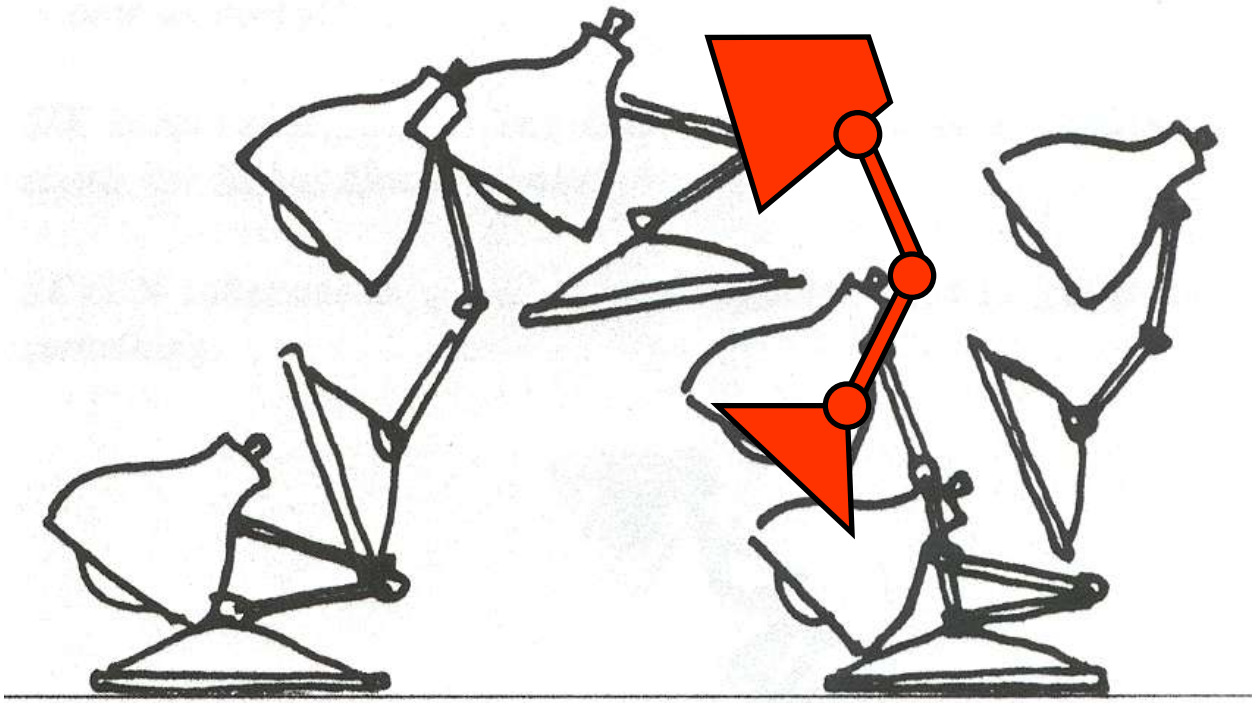
- Define character poses at specific time steps called “keyframes”



Lasseter `87

Keyframe Animation

- Interpolate variables describing keyframes to determine poses for character “in-between”



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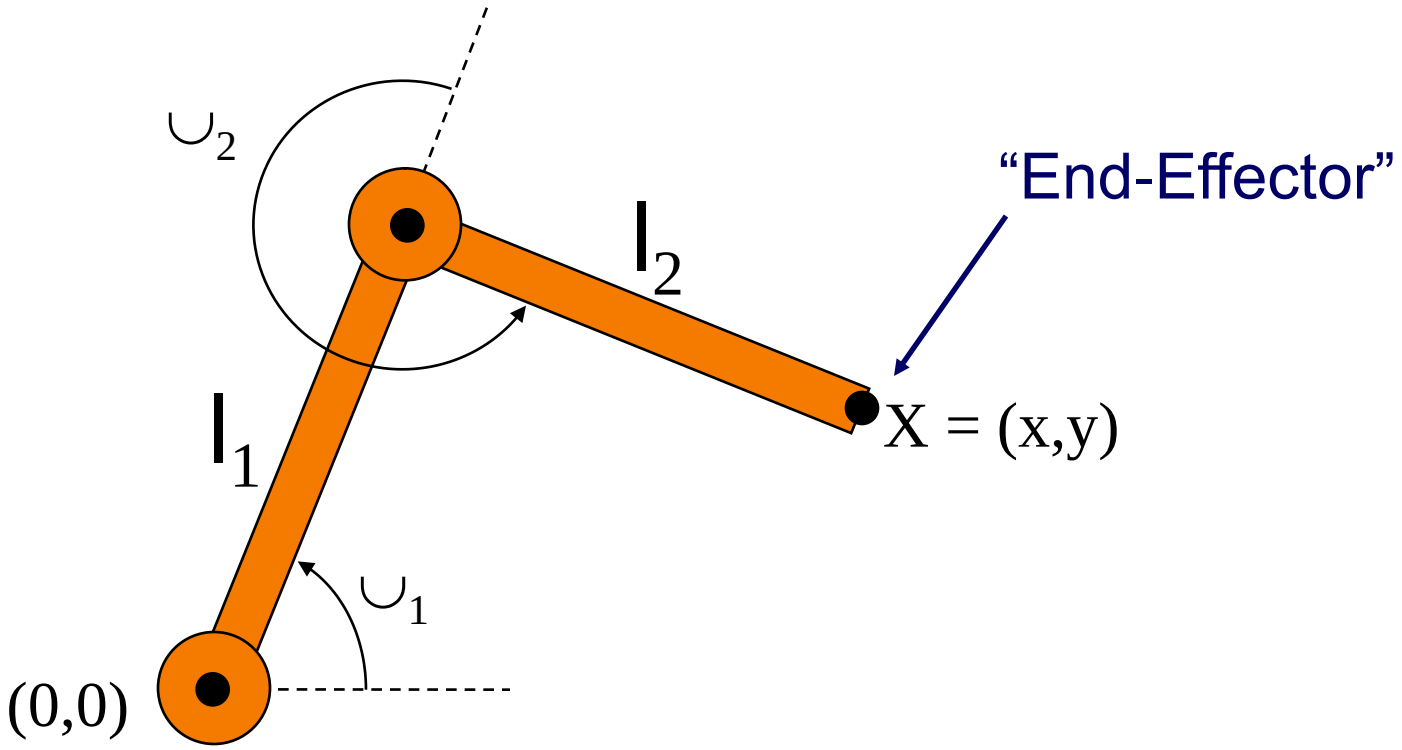
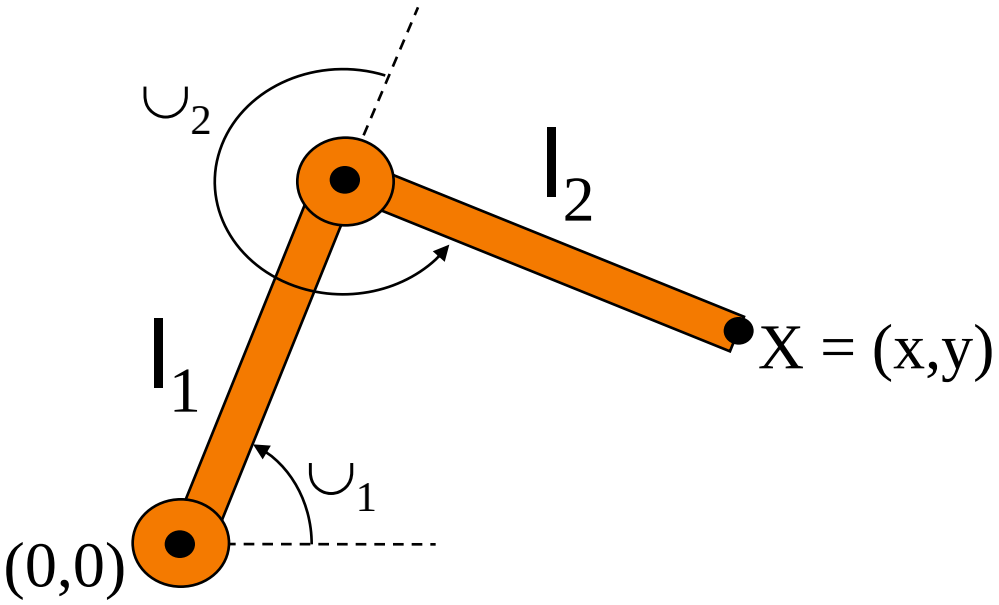
Example: 2-Link Structure

- Two links connected by rotational joints



Forward Kinematics

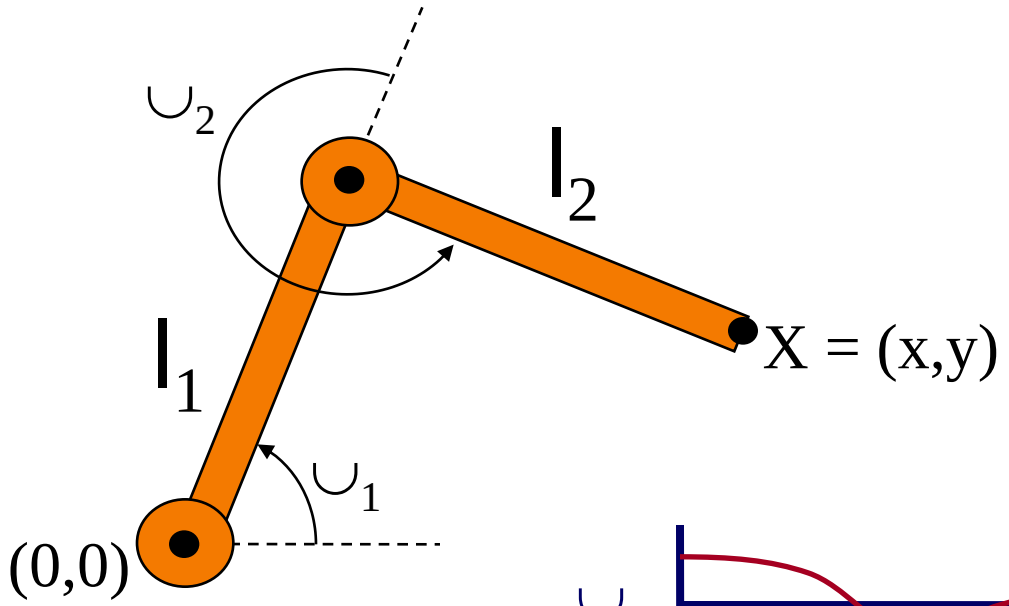
- Animator specifies joint angles: $\cup_1$ and $\cup_2$
- Computer finds positions of end-effector: X

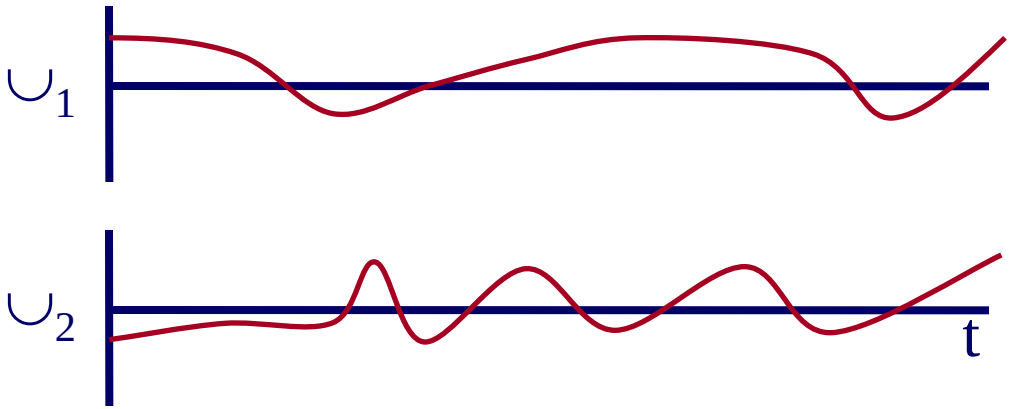


$$X = (l_1 \cos\Theta_1 + l_2 \cos(\Theta_1 + \Theta_2), l_1 \sin\Theta_1 + l_2 \sin(\Theta_1 + \Theta_2))$$

Forward Kinematics

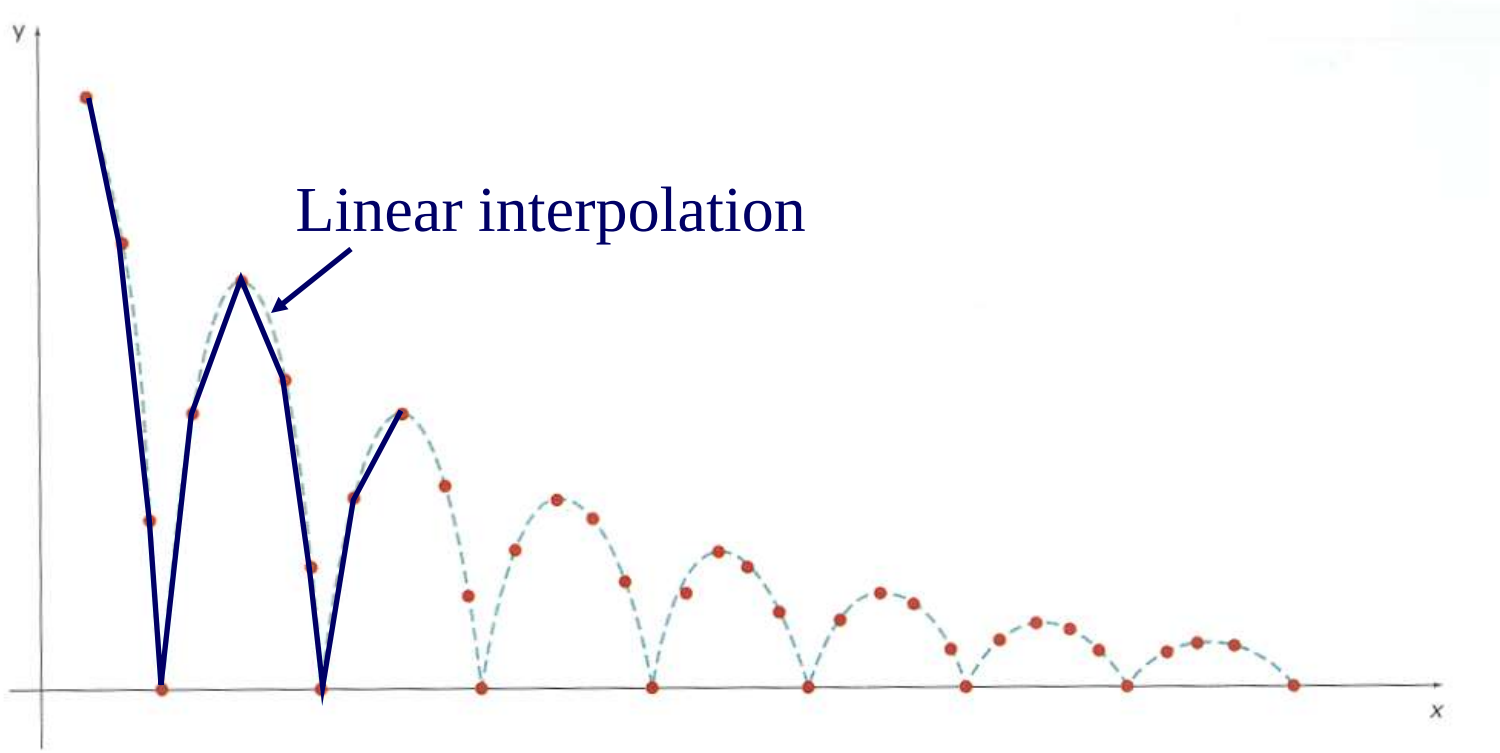
- Joint motions can be specified by spline curves





Keyframe Animation

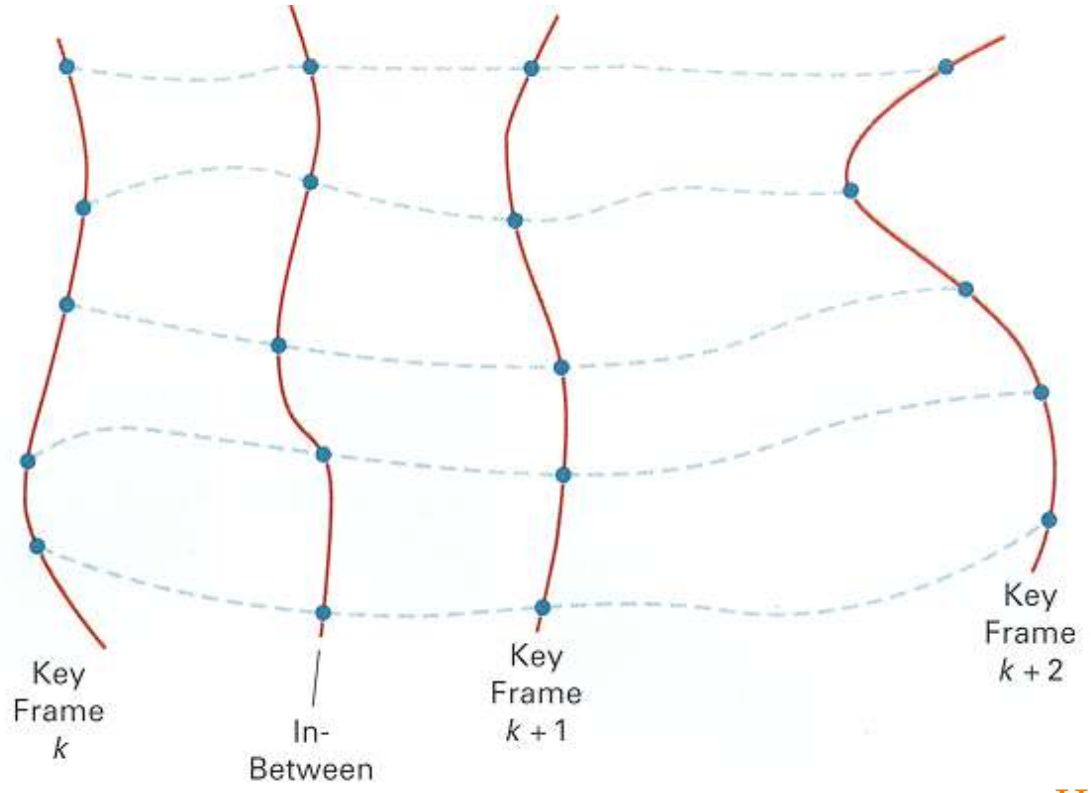
- Inbetweening:
  - Linear interpolation - usually not enough continuity



H&B Figure 16.16

Keyframe Animation

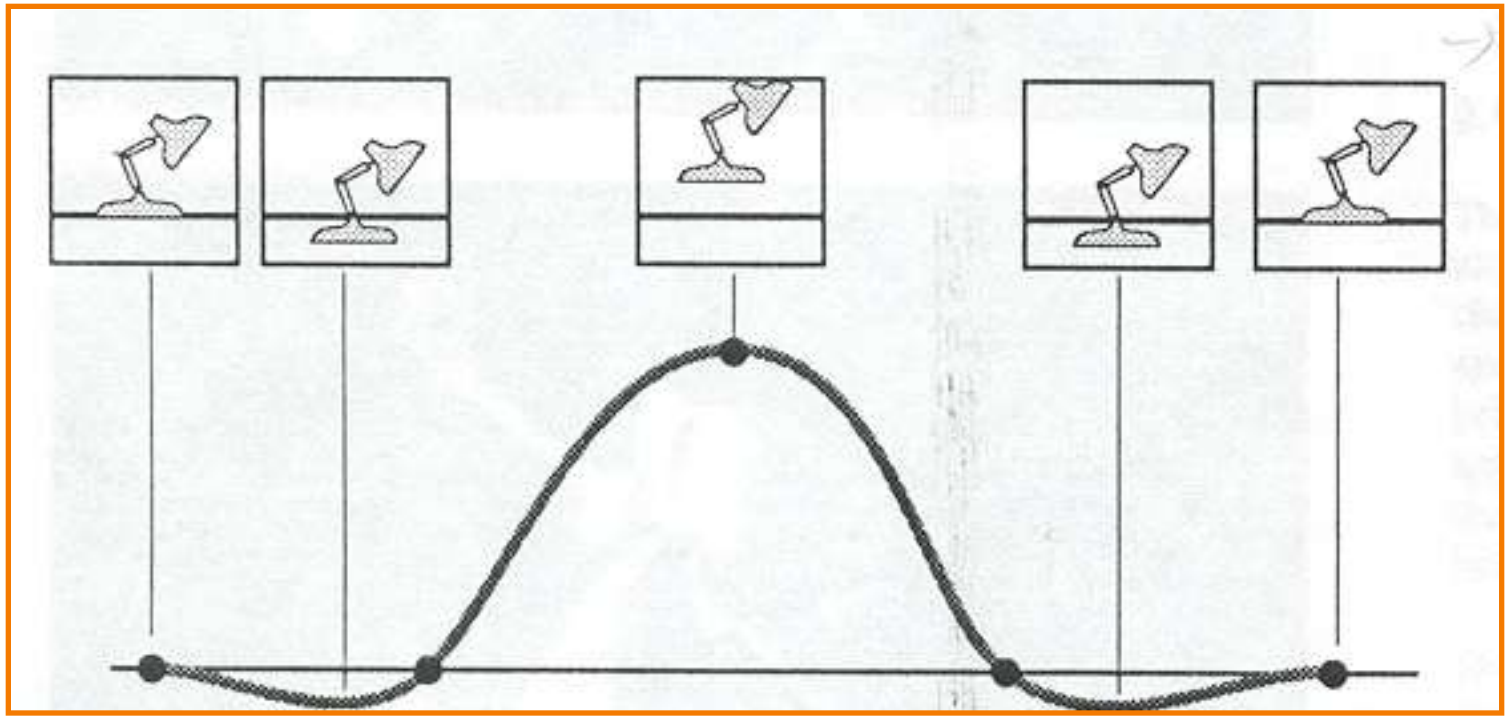
- Inbetweening:
  - Spline interpolation - maybe good enough



H&B Figure 16.11

Keyframe Animation

- Inbetweening:
  - Cubic spline interpolation - maybe good enough
    - » May not follow physical laws



Lasseter `87

Keyframe Animation

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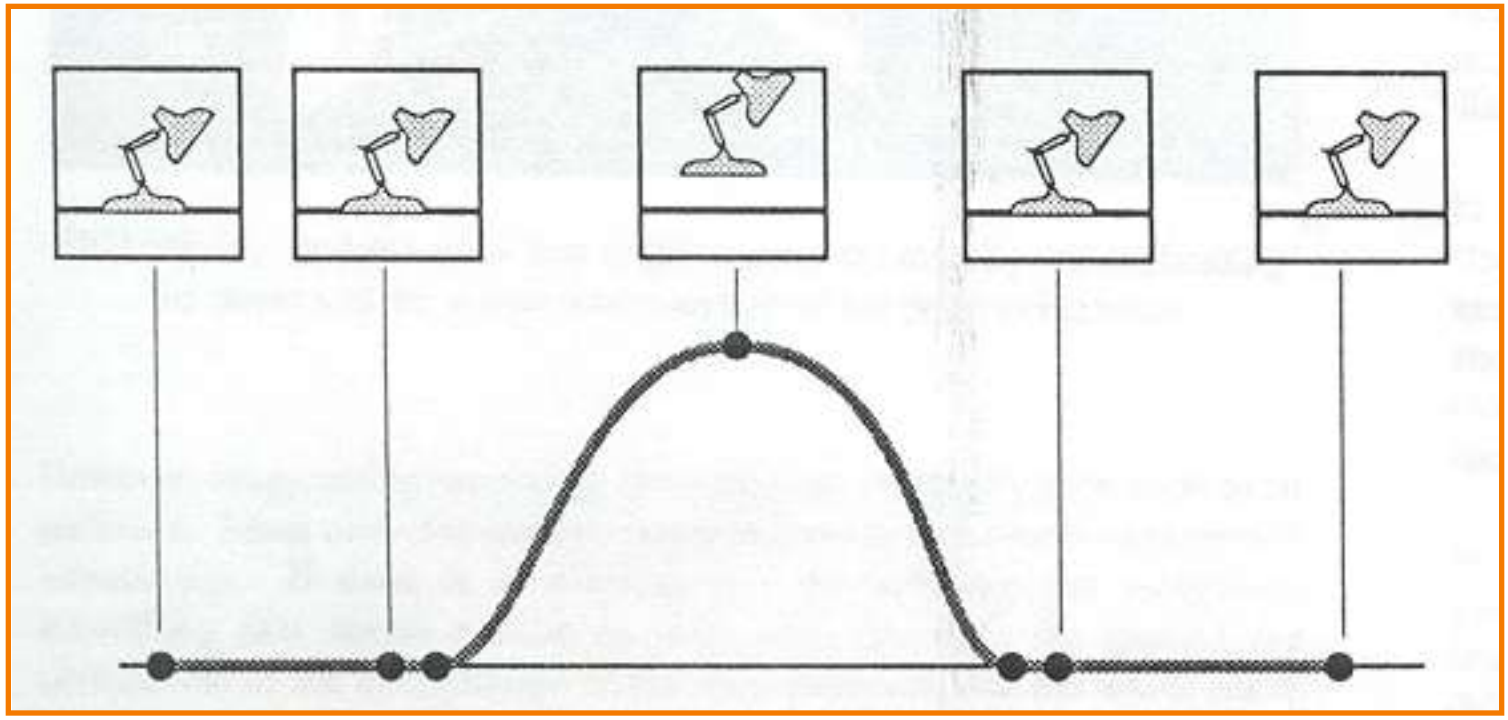

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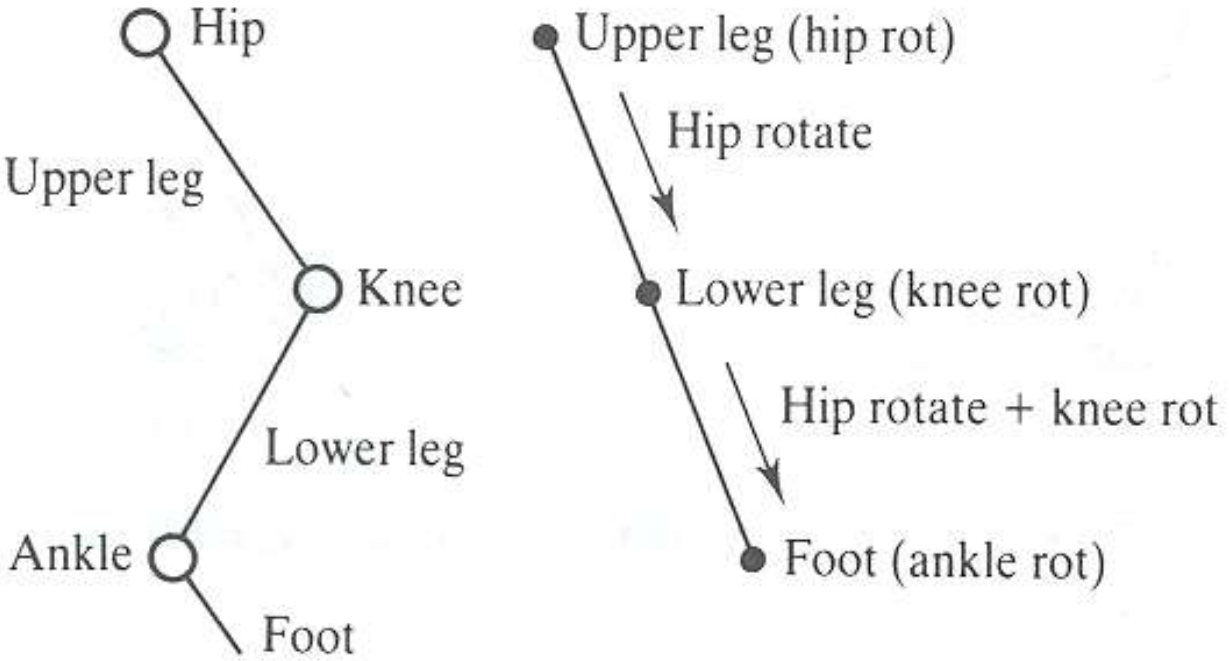
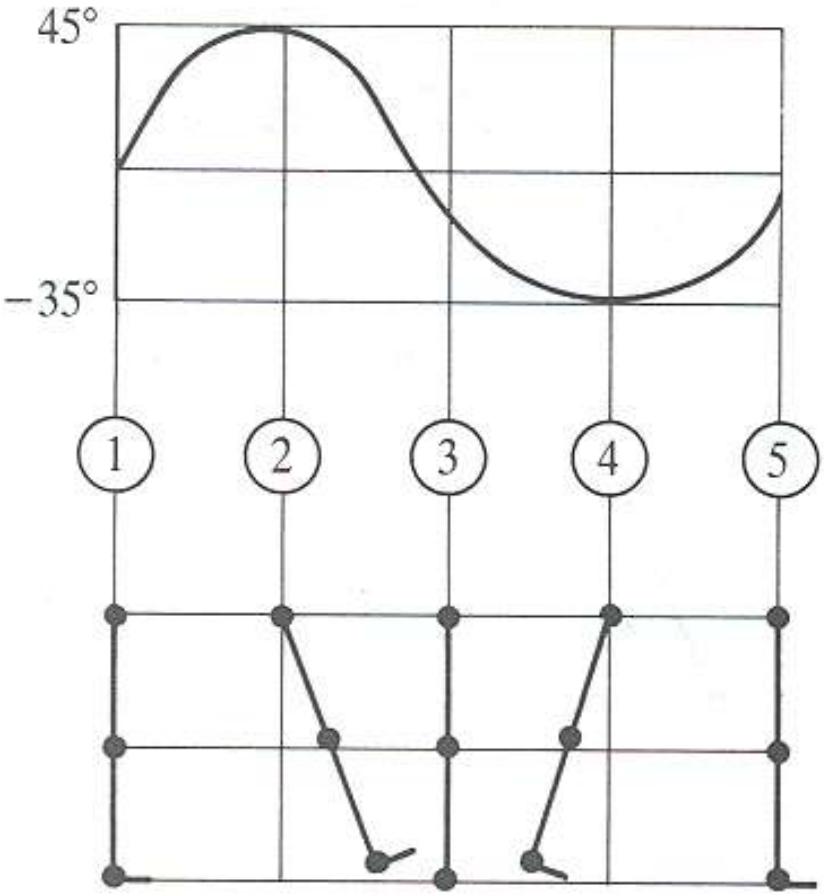
Example: Walk Cycle

- Articulated figure:



Example: Walk Cycle

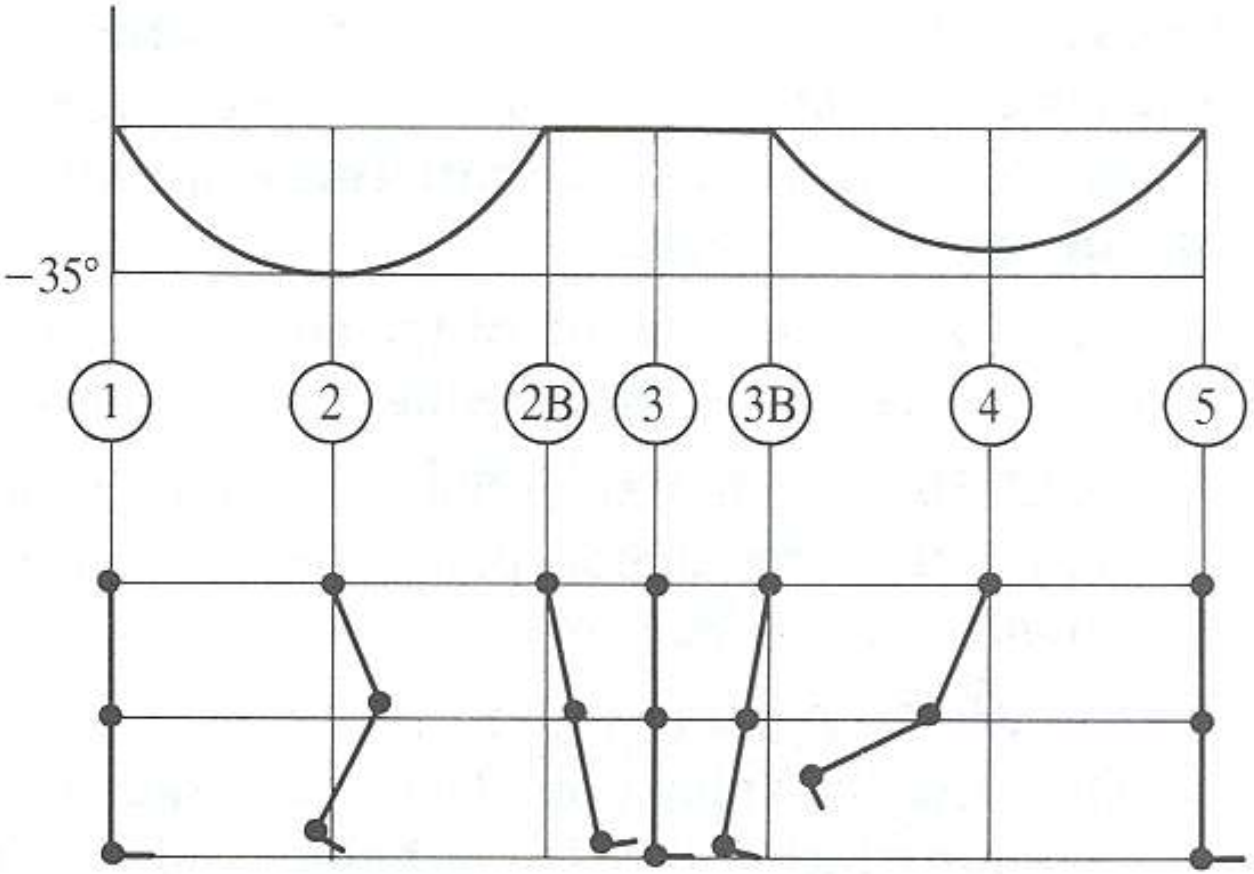
- Hip joint orientation:



Watt & Watt

Example: Walk Cycle

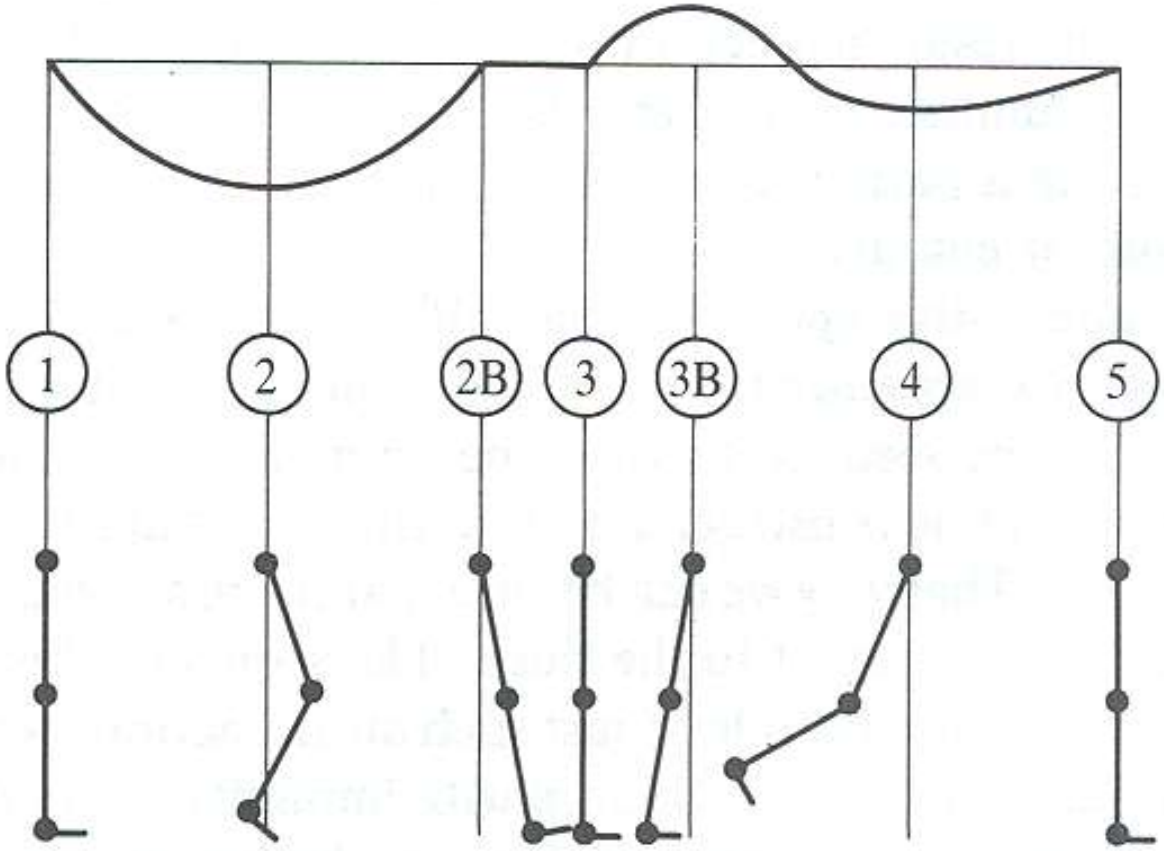
- Knee joint orientation:



Watt & Watt

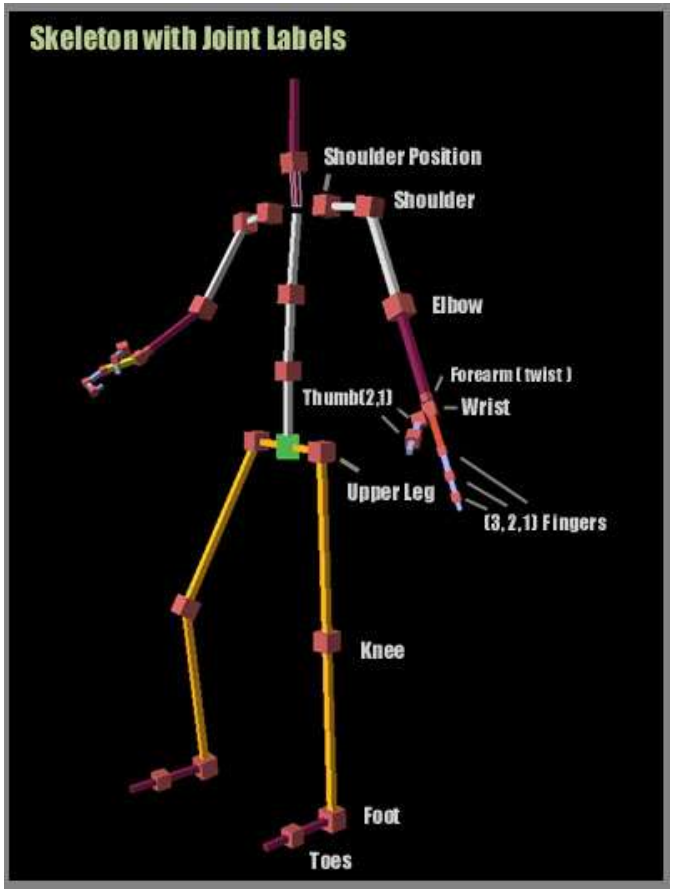
Example: Walk Cycle

- Ankle joint orientation:



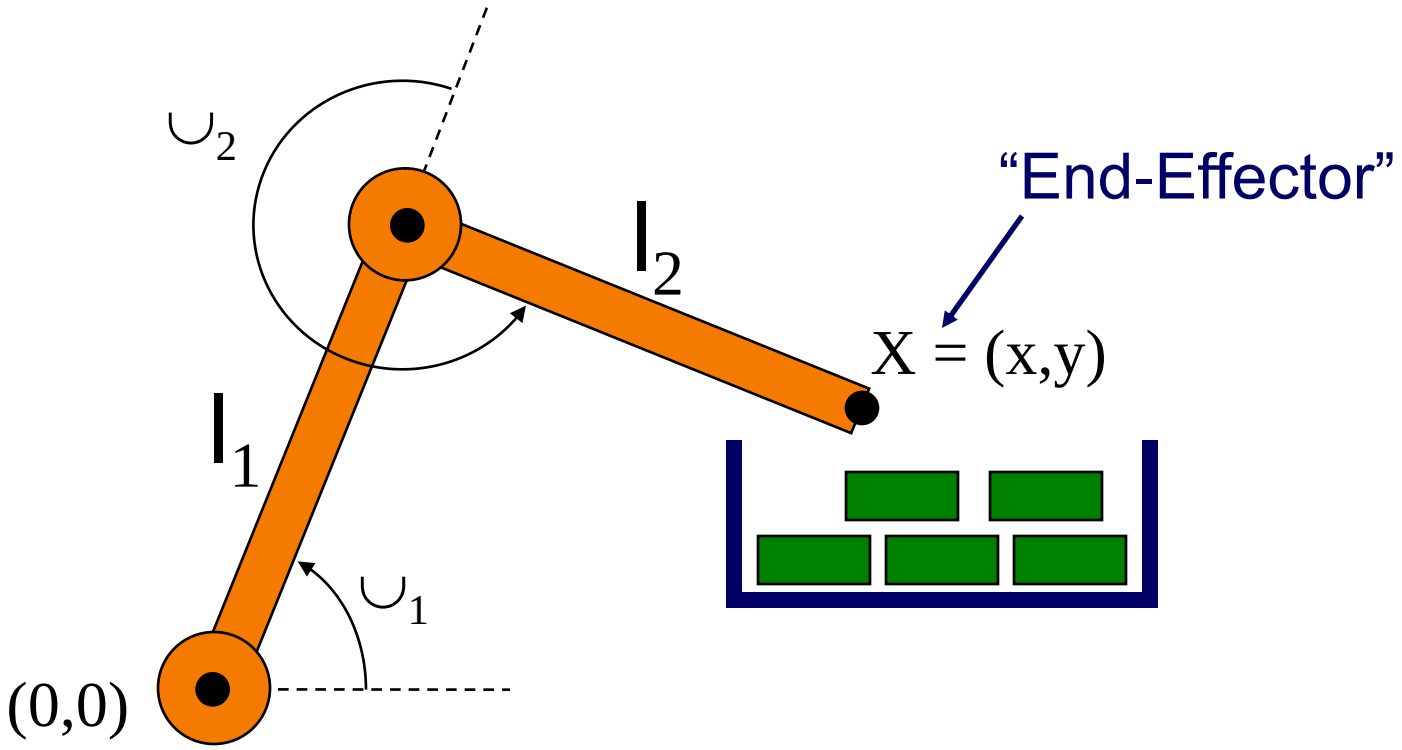
Outline

- Keyframe animation
- Adding inverse kinematics
- Adding dynamics



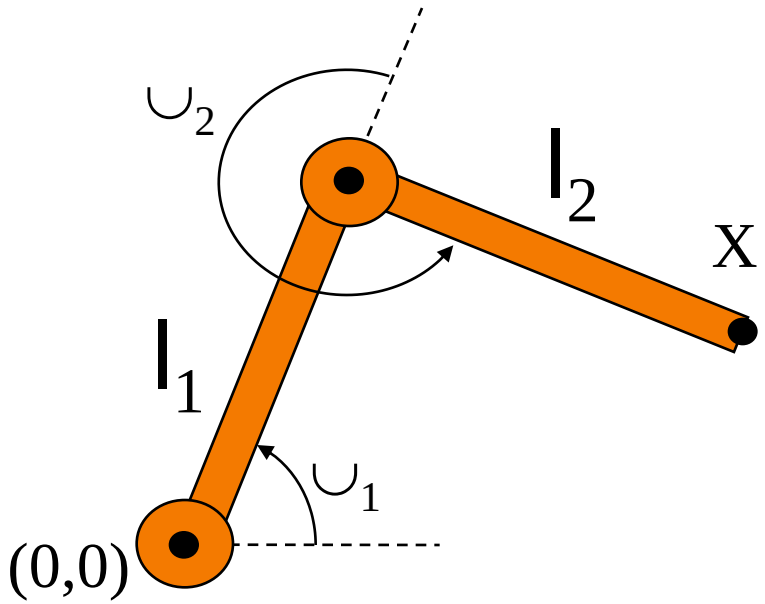
Example: 2-Link Structure

- What if animator knows position of “end-effector”



Inverse Kinematics

- Animator specifies end-effector positions: X
- Computer finds joint angles: $\cup_1$ and $\cup_2$:

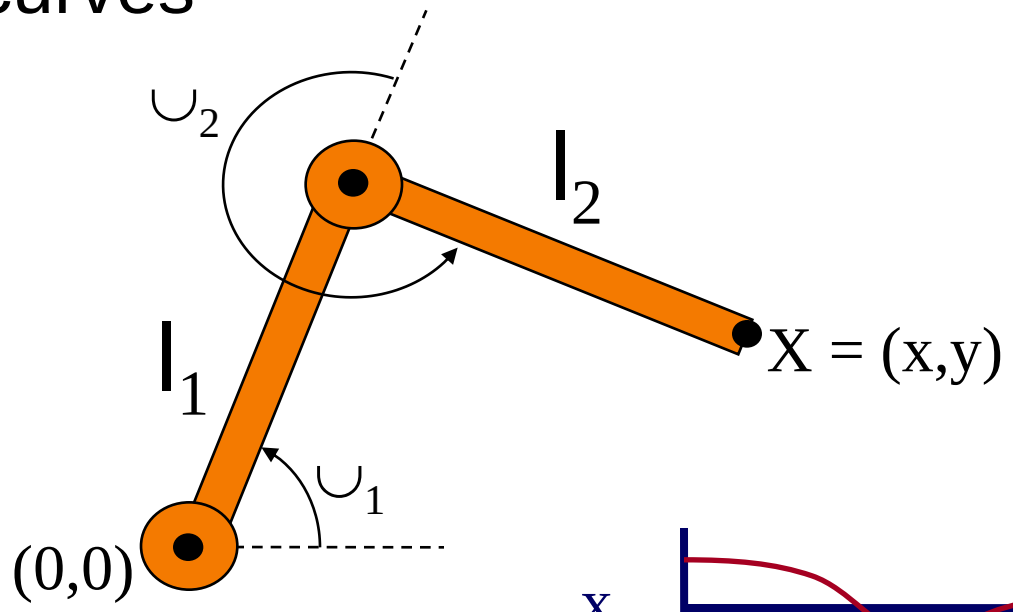


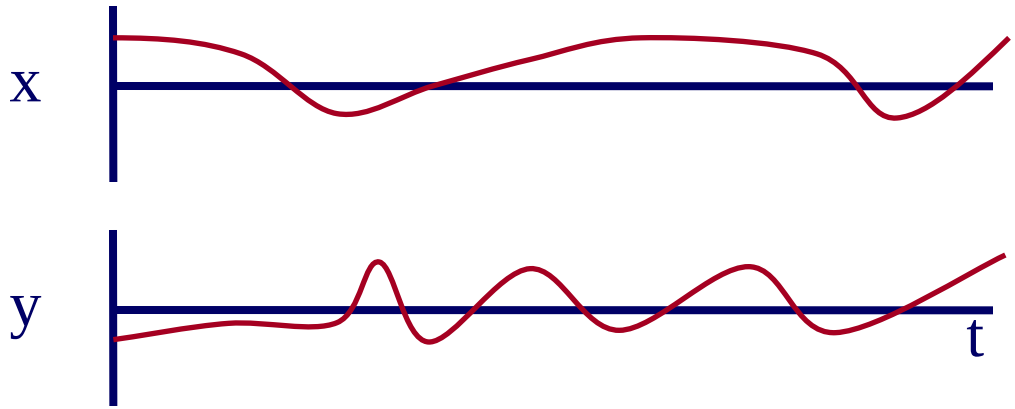
$$\Theta_2 = \cos^{-1}\left(\frac{x^2 + x^2 - l_1^{\,2} - l_2^{\,2}}{2 l_1 l_2}\right)$$

$$\Theta_1 = \frac{-(l_2 \sin(\Theta_2)x + (l_1 + l_2 \cos(\Theta_2))y}{(l_2 \sin(\Theta_2))y + (l_1 + l_2 \cos(\Theta_2))x}$$

Inverse Kinematics

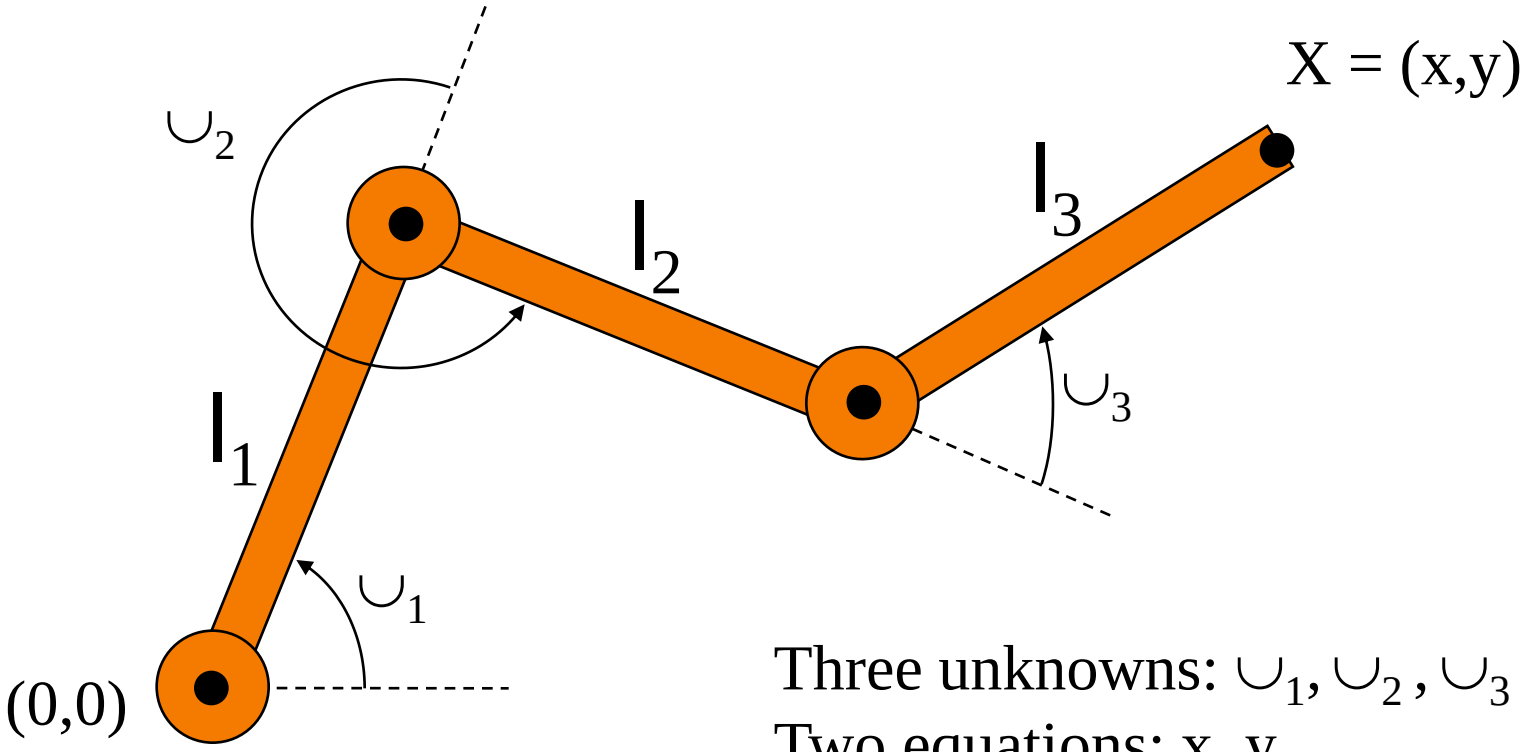
- End-effector postions can be specified by spline curves





Inverse Kinematics

- Problem for more complex structures
  - System of equations is usually under-defined
  - Multiple solutions

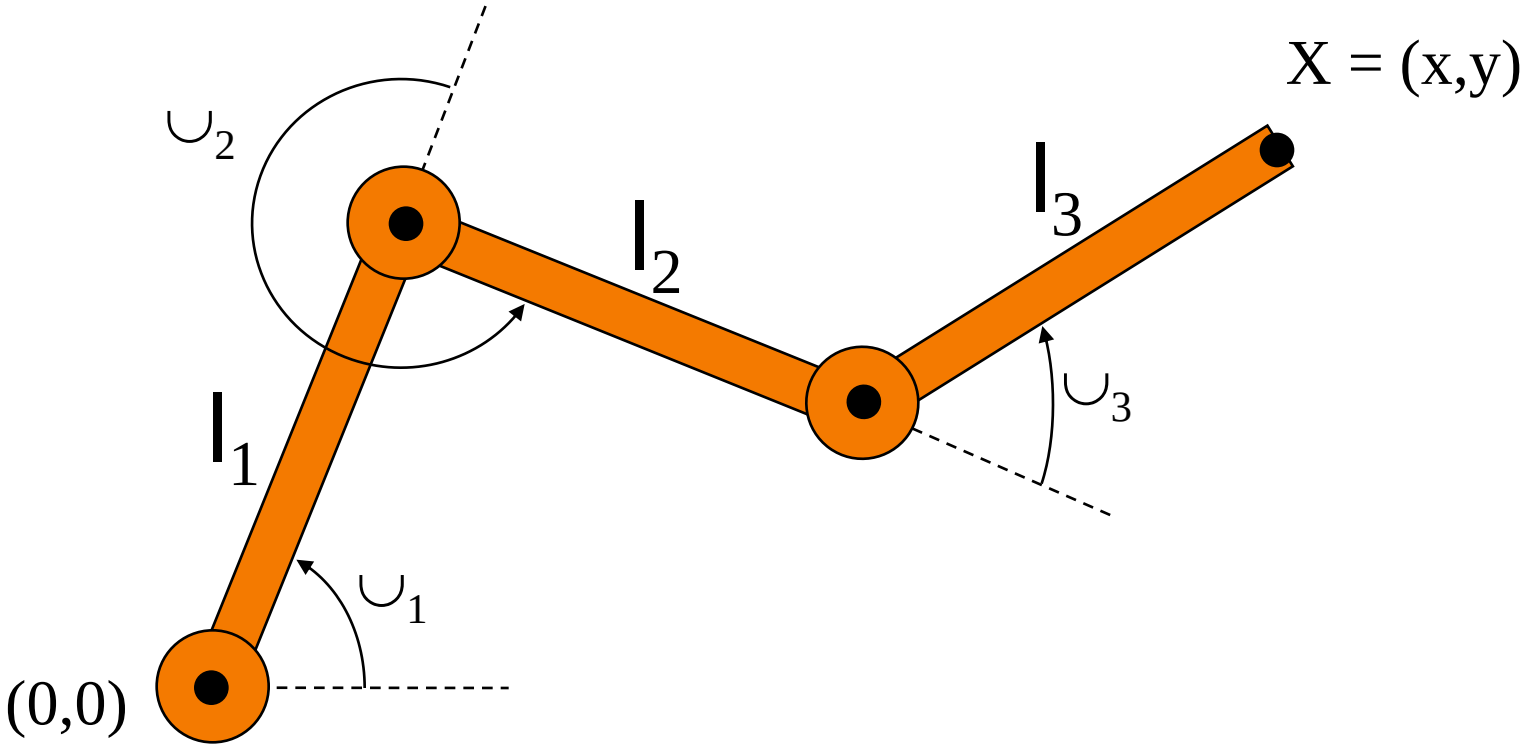


Three unknowns: $\cup_1, \cup_2, \cup_3$

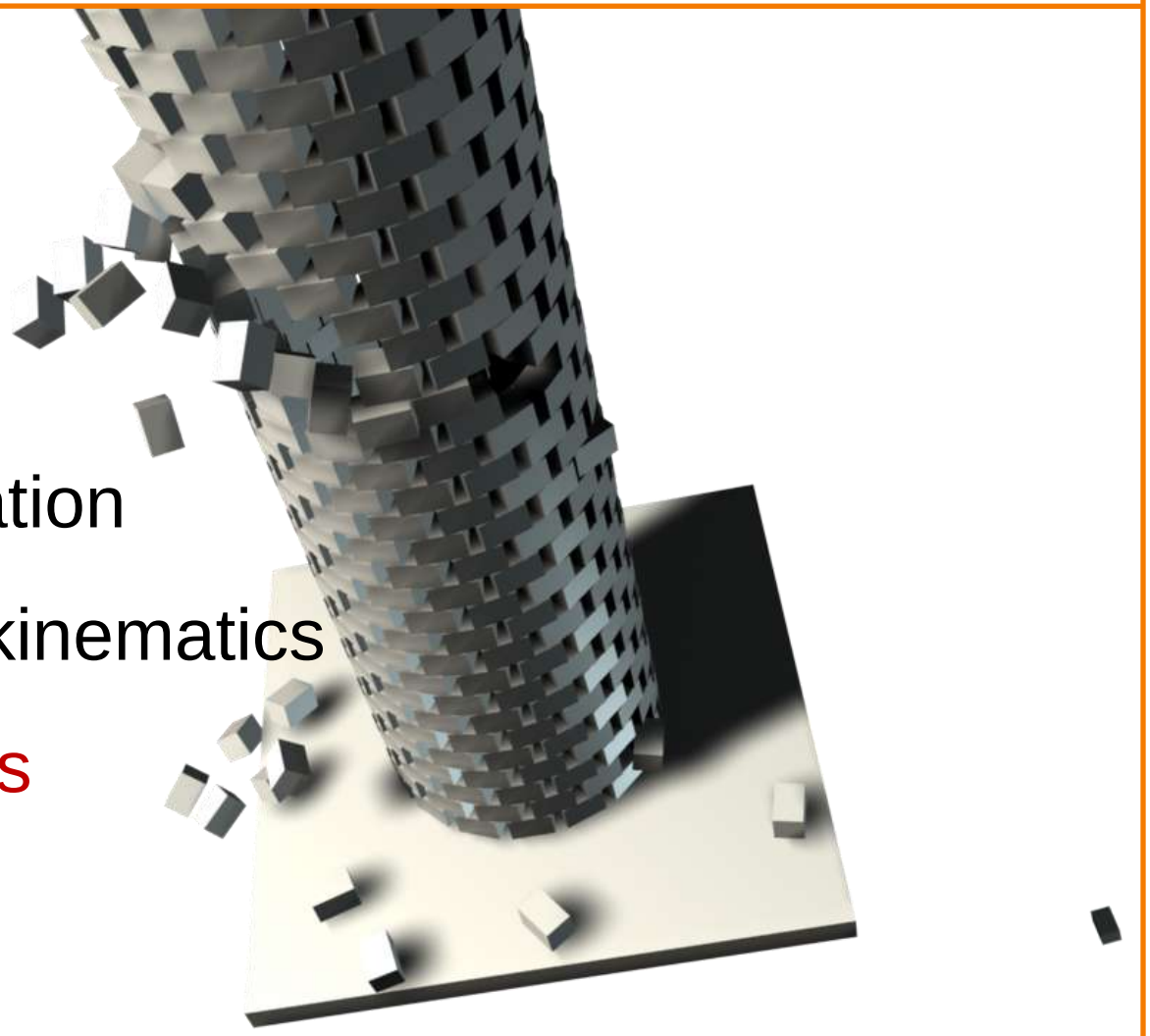
Two equations: x, y

Inverse Kinematics

- Solution for more complex structures:
  - Find best solution (e.g., minimize energy in motion)
  - Non-linear optimization



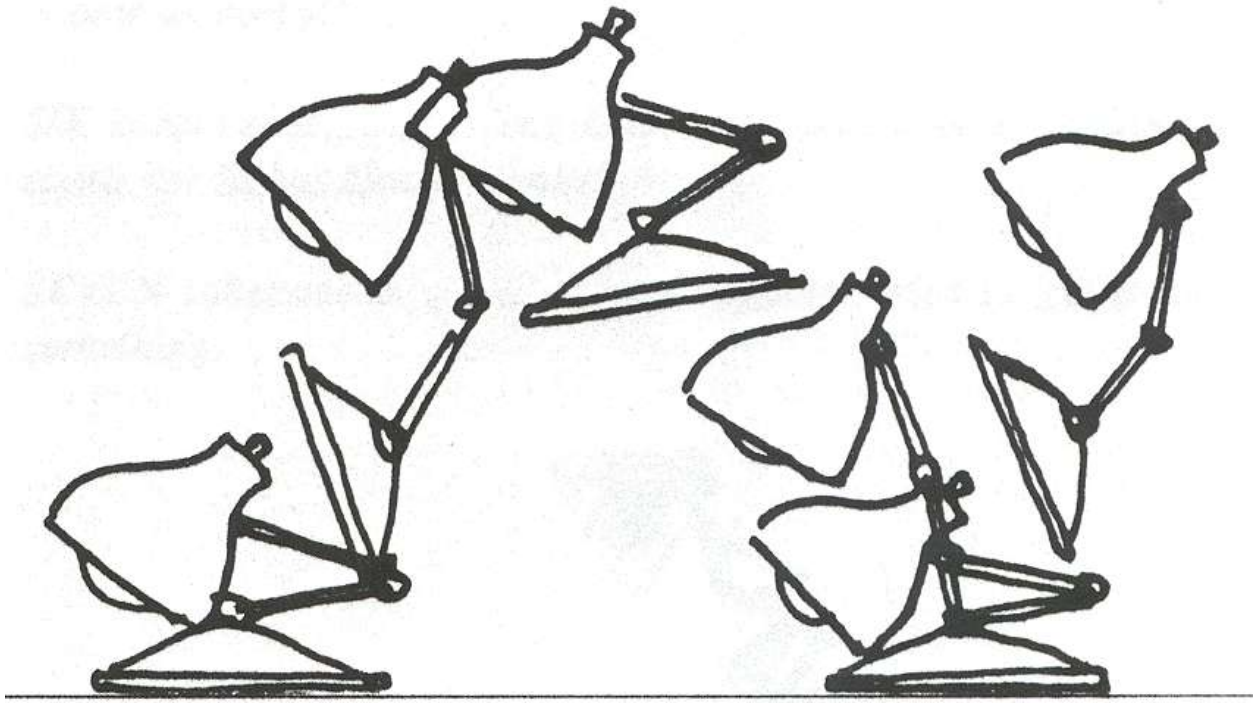
Outline



- Keyframe animation
- Adding inverse kinematics
- Adding dynamics

Dynamics

- Simulation of physics insures realism of motion



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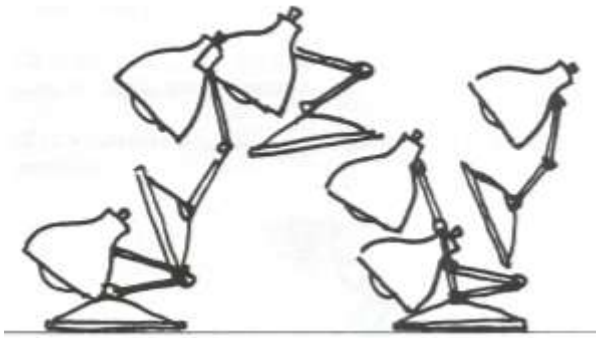
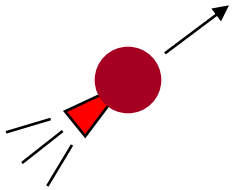
Spacetime Constraints

- Animator specifies constraints:
  - What the character's physical structure is
    - » e.g., articulated figure
  - What the character has to do (keyframes)
    - » e.g., jump from here to there within time t
  - What other physical structures are present
    - » e.g., floor to push off and land
  - How the motion should be performed
    - » e.g., minimize energy



Spacetime Constraints

- Computer finds the “best” physical motion satisfying constraints
- Example: particle with jet propulsion
  - **x**(t) is position of particle at time t
  - **f**(t) is force of jet propulsion at time t
  - Particle’s equation of motion is:



$$mx'' - f - mg = 0$$

  - Suppose we want to move from a to b within t_0 to t_1 with minimum jet fuel:

$$\text{Minimize} \int_{t_0}^{t_1} \left| f(t)^2 \right| dt \quad \text{subject to } x(t_0)=a \text{ and } x(t_1)=b$$

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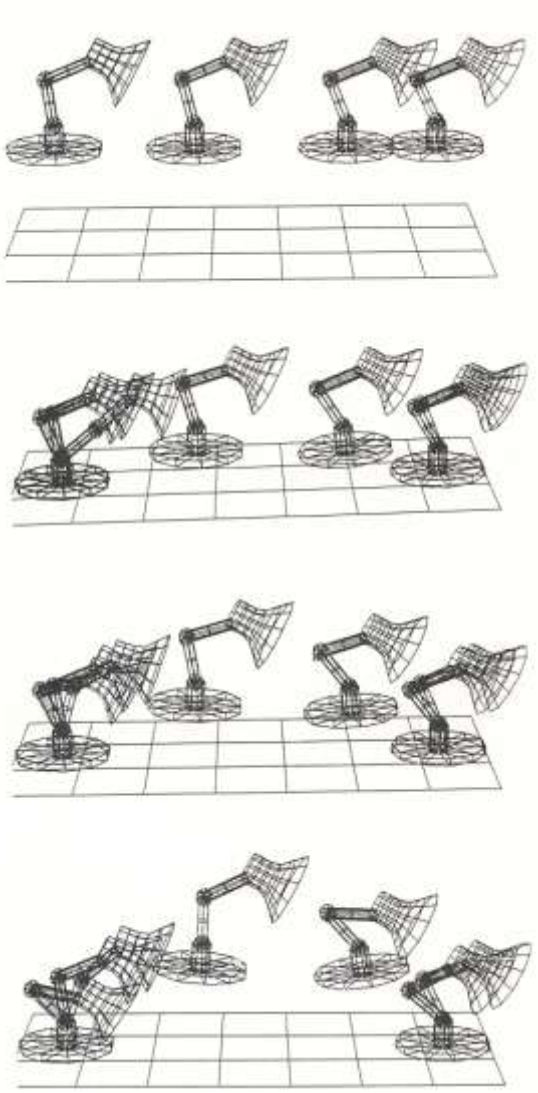
Spacetime Constraints

- Solve with iterative optimization methods



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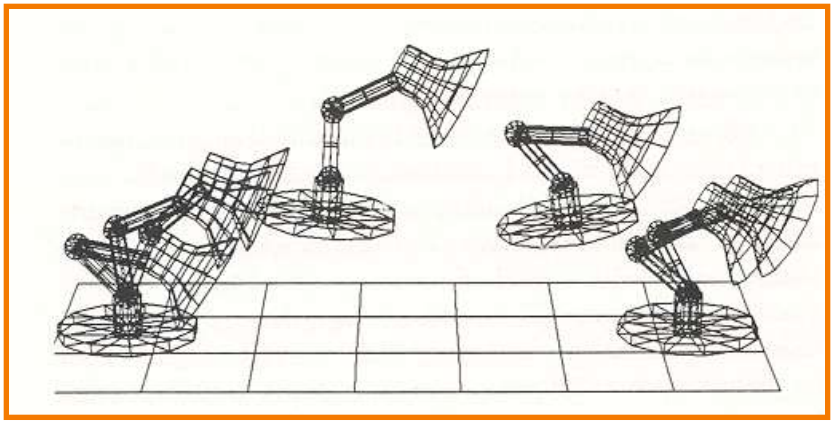
Spacetime Constraints

- Advantages:
  - Free animator from having to specify details of physically realistic motion with spline curves
  - Easy to vary motions due to new parameters and/or new constraints

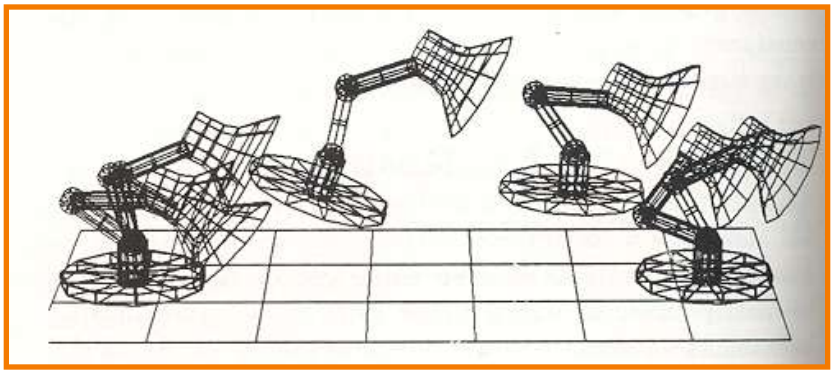
- Challenges:
  - Specifying constraints and objective functions
  - Avoiding local minima during optimization

Spacetime Constraints

- Adapting motion:



Original Jump



Heavier Base

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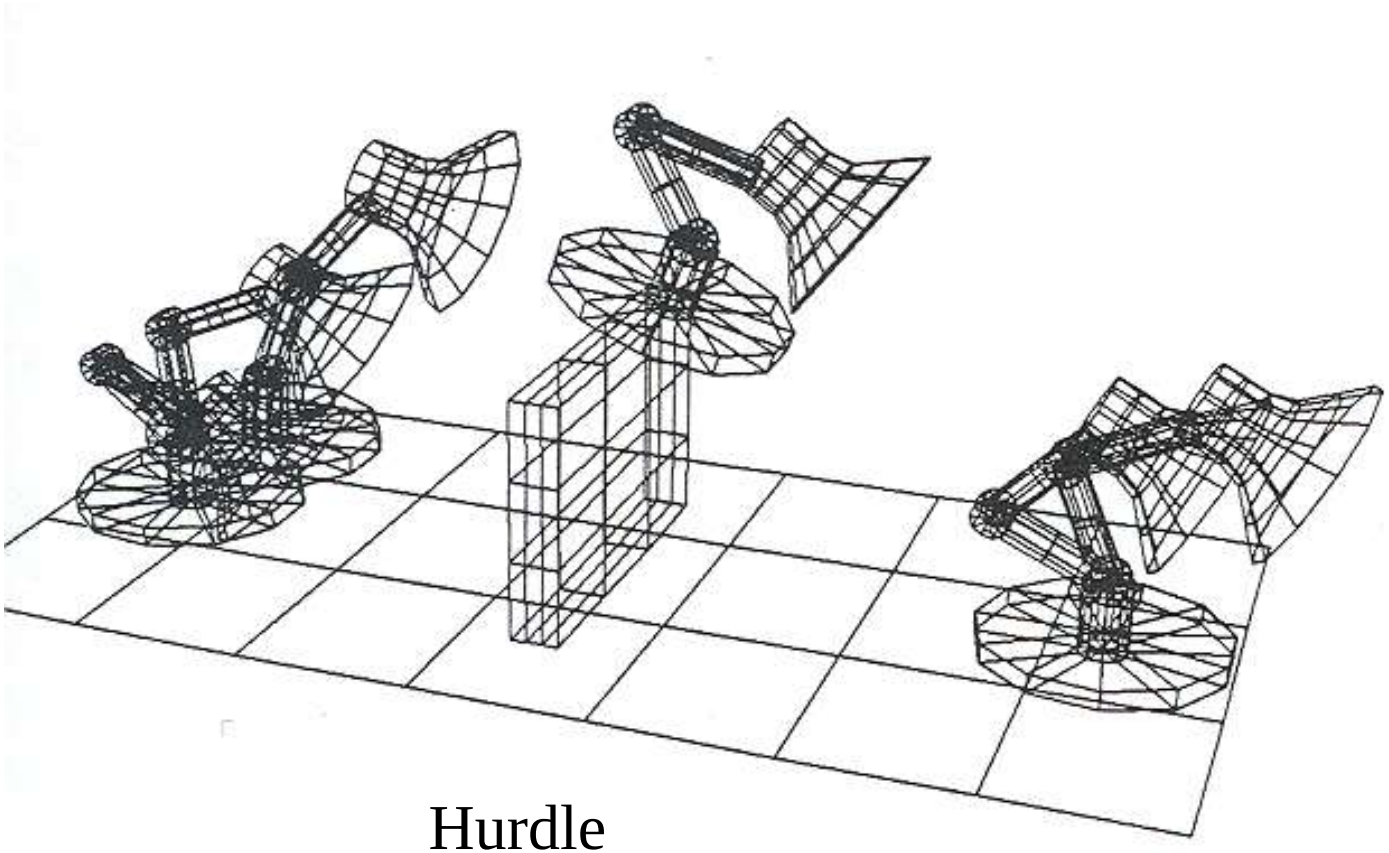
Spacetime Constraints

- Adapting motion:



Hurdle

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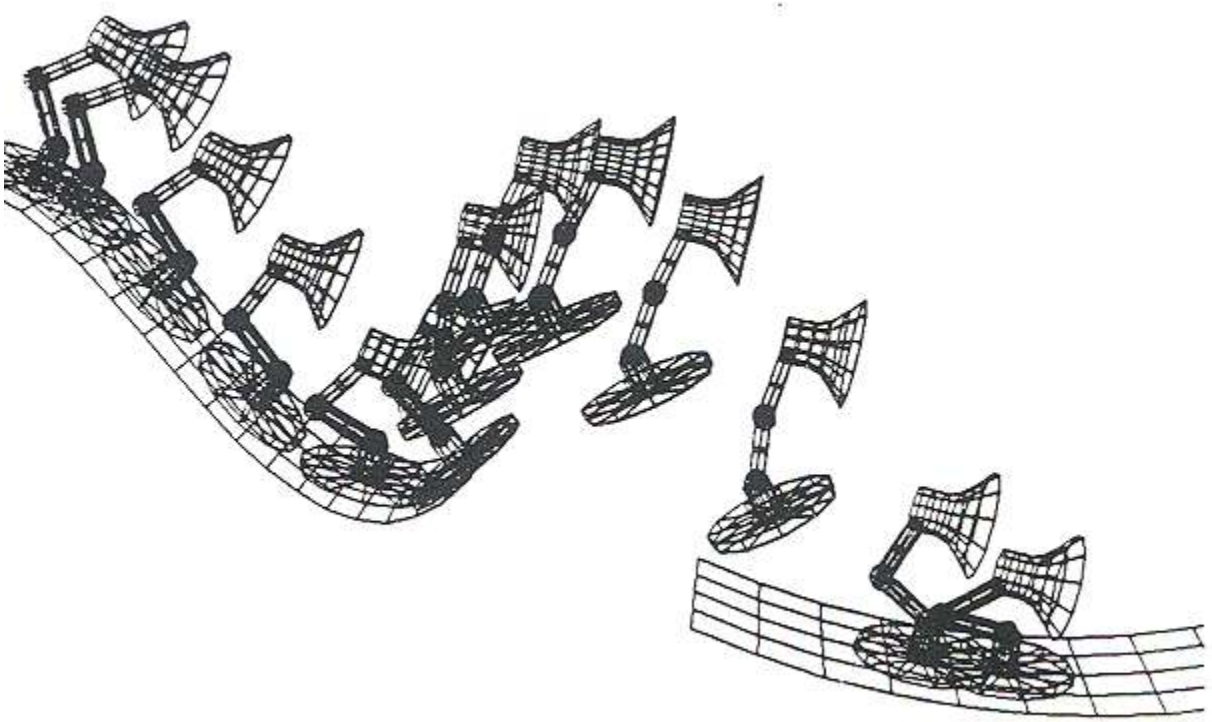
Spacetime Constraints

- Adapting motion:



Ski Jump

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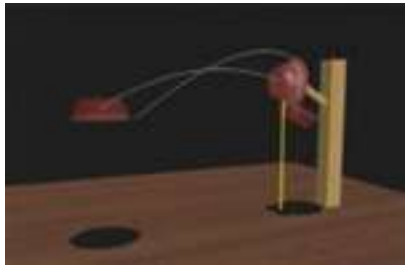
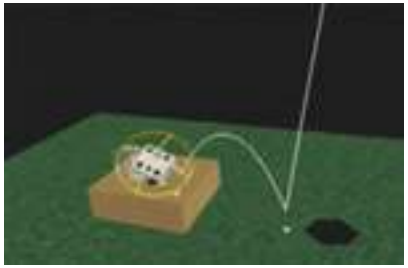
- Challenges:
  - Specifying constraints and objective functions
  - Avoiding local minima during optimization

Example: Manipulation of Sims.

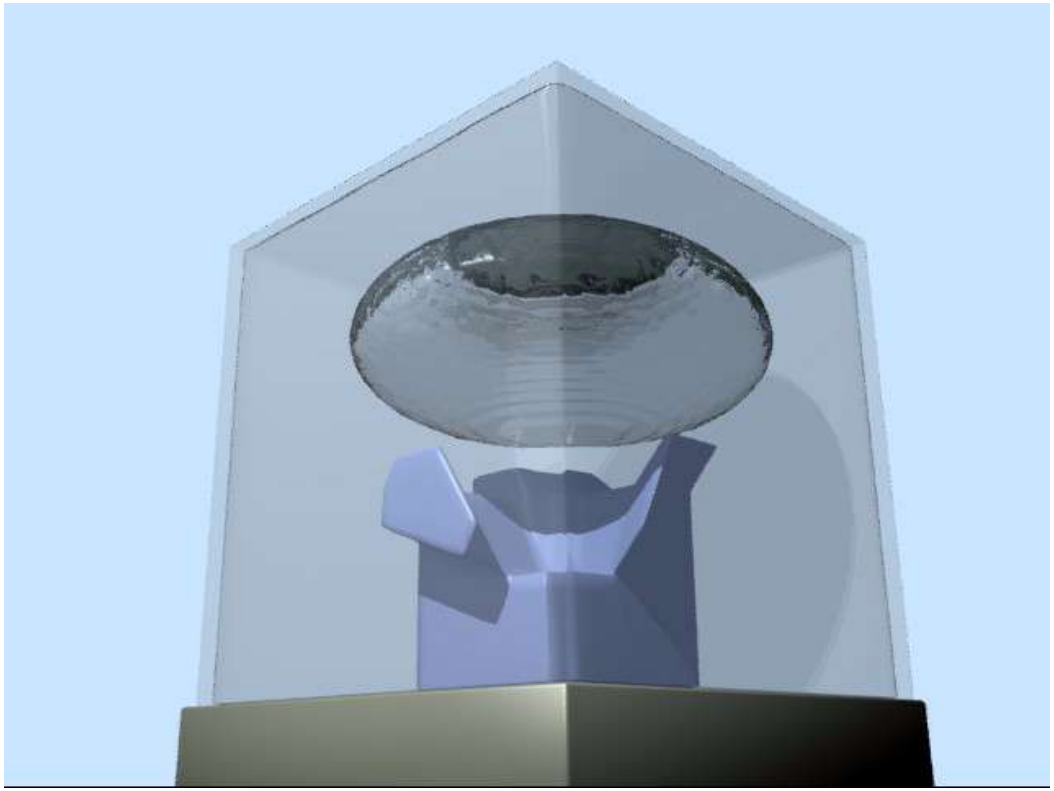




Interactive Manipulation of Rigid Body Simulations.
Popovic et al Siggraph 2000.

Summary

- Keyframe animation
  - Poses specified at key times
  - In-betweening to fill in the rest
- Incorporating inverse kinematics
  - Makes keyframes easier to specify
- Incorporating dynamics
  - Makes animation easier to adapt

Simulation

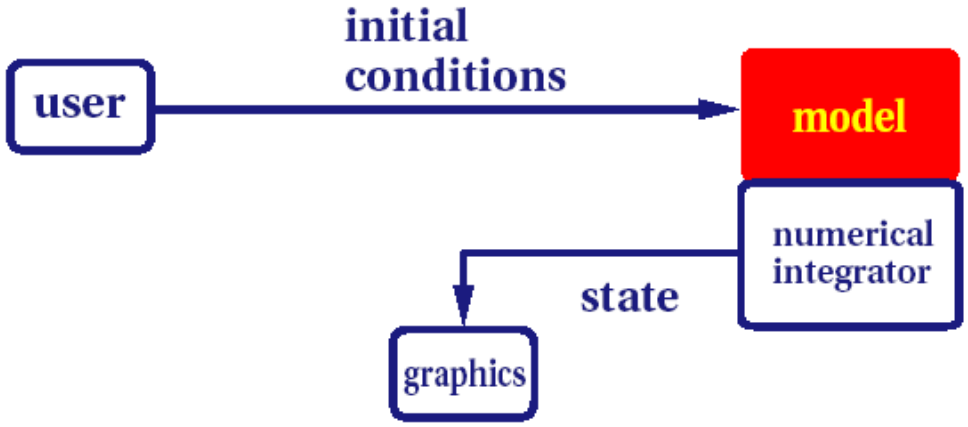


Simulation

- Dynamics
  - Considers underlying forces
  - Compute motion from initial conditions and physics
- Kinematics
  - Considers only motion
  - Determined by positions, velocities, accelerations

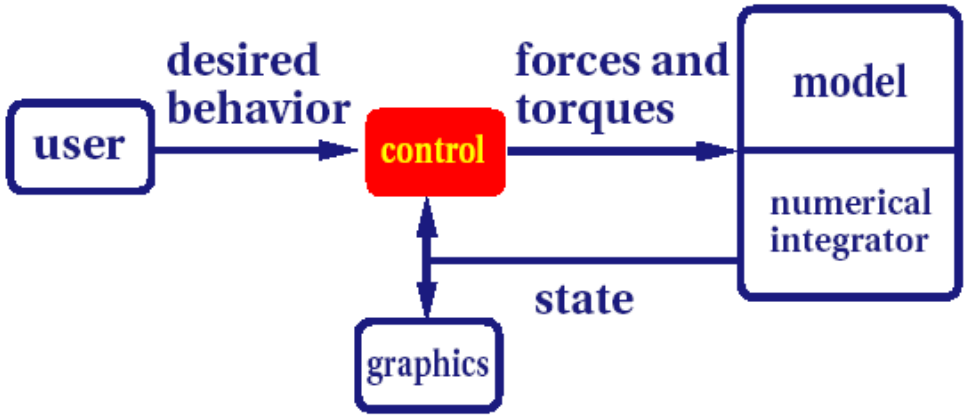
Dynamics

Passive--no muscles or motors



particle systems
leaves
water spray
clothing

Active--internal source of energy



running human
trotting dog
swimming fish

Hodgins

Passive Dynamics

- No muscles or motors
  - Smoke
  - Water
  - Cloth
  - Fire
  - Fireworks
  - Dice

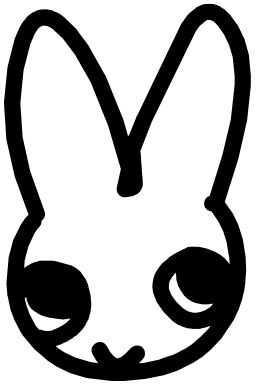
Passive Dynamics

- Physical laws
  - Newton's laws
  - Hook's law
  - Etc.
- Physical phenomena
  - Gravity
  - Momentum
  - Friction
  - Collisions
  - Elasticity
  - Fracture

Fun with Bunny

Porous Flow in Particle-Based Fluid Simulations
Lenaerts et al, SIGGRAPH 2007

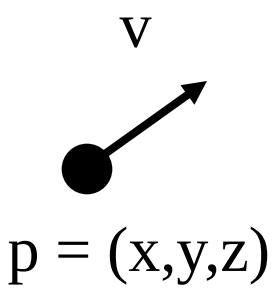
Fire with cellular patterns
Ron Fedkiw



A Finite Element Method for Animating Large Viscoplastic Flow
Bargteil et al SIGGRAPH 2007

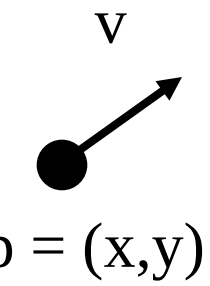
Particle Systems

- A particle is a point mass
  - Mass
  - Position
  - Velocity
  - Forces
  - Color
  - Lifetime



- Use lots of particles to model complex phenomena
  - Keep array of particles
  - Newton’s laws

Our Particle




```
enum ParticleType { Create,Update };

struct Vector2d{
   double x;
   double y;

};

struct Particle {
   Vector2d Pos; //Position of the particle
   Vector2d Vel; //Velocity of the particle
   int age; //Current age of the particle
   int LifeSpan; //Age after which the particle dies
   int color;
   int size;

};
```

Particle Systems

- For each frame:
  - Create new particles and assign attributes
  - Delete any expired particles
  - Update particles based on attributes and physics
  - Render particles

Our Particle

```
void Init(long num_part,long num_forces, Vector2d Forces[],
   ParticleType part_type,Vector2d vel, Vector2d pos1,Vector2d
   pos2, int lifespan, int color, int size){

Particles = new Particle[num_part];
   ParticleNum=num_part;

for(int i=0;i<num_forces;i++){
       TotForce.x+=Forces[i].x;
       TotForce.y+=Forces[i].y;

}
   Particle_Type=part_type;
   Vel=vel;
   Pos1=pos1;
   Pos2=pos2;
   LifeSpan=lifespan;
   Color=color;
   Size=size;
   randomize();
   InitParticles();

}
```

Creating Particles

- Where to create particles?
  - ➢ Predefined source
  - ◦ Surface of shape
  - ◦ Where particle density is low
  - ◦ etc.

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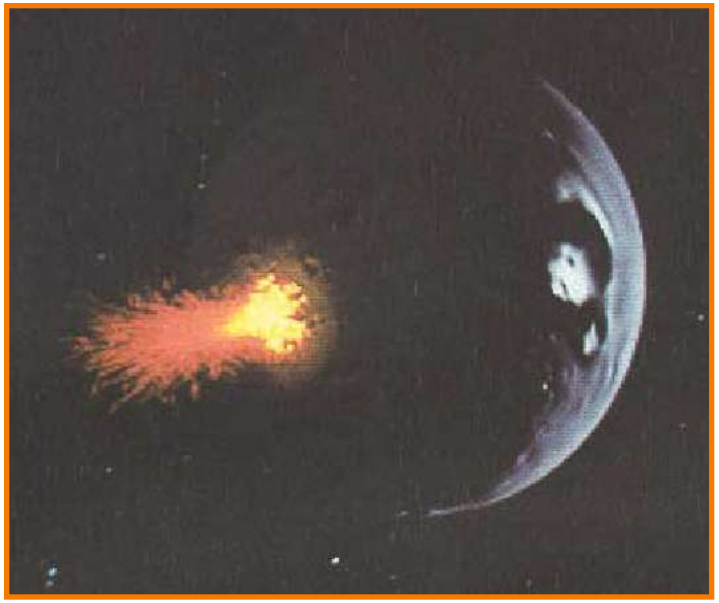
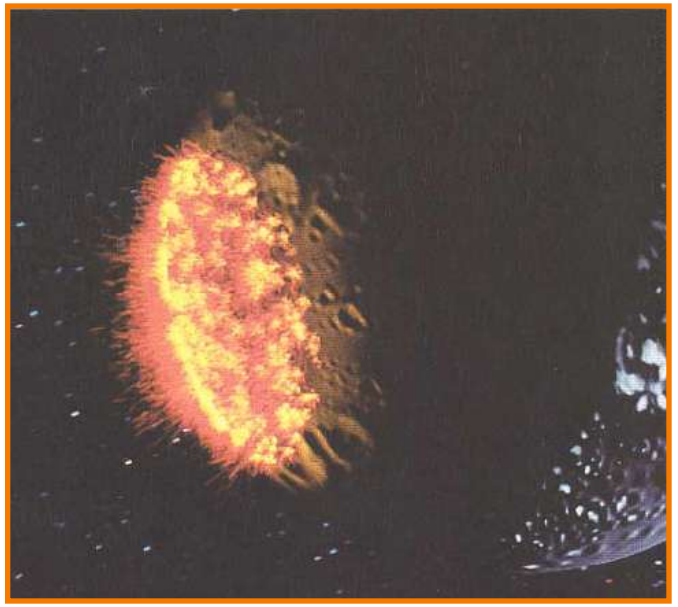
Creating Particles

- Where to create particles?
  - Predefined source
  - ➢ Surface of shape
  - Where particle density is low
  - etc.

Reeves





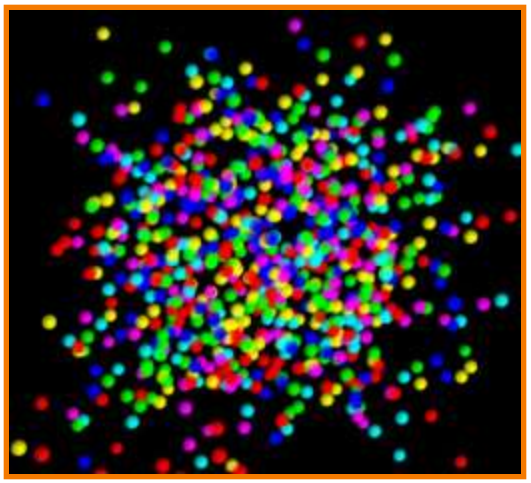
Deleting Particles

- When to delete particles?
  - Predefined sink
  - Surface of shape
  - Where density is high
  - Life span
  - Random

McAllister

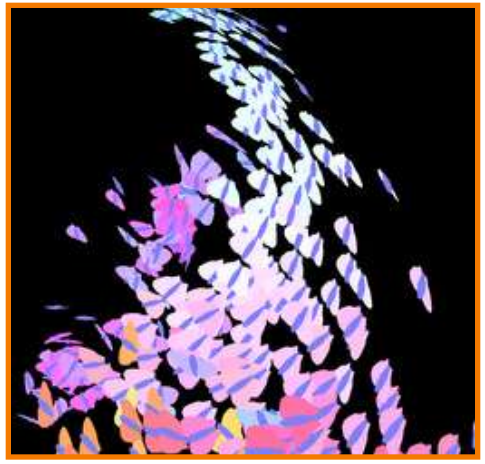
Rendering Particles

- Rendering styles
  - ➢ Points
  - ◦ Polygons
  - ◦ Shapes
  - ◦ Trails
  - ◦ etc.



Rendering Particles

- Rendering styles
  - Points
  - ➢ Polygons
  - Shapes
  - Trails
  - etc.



Rendering Particles

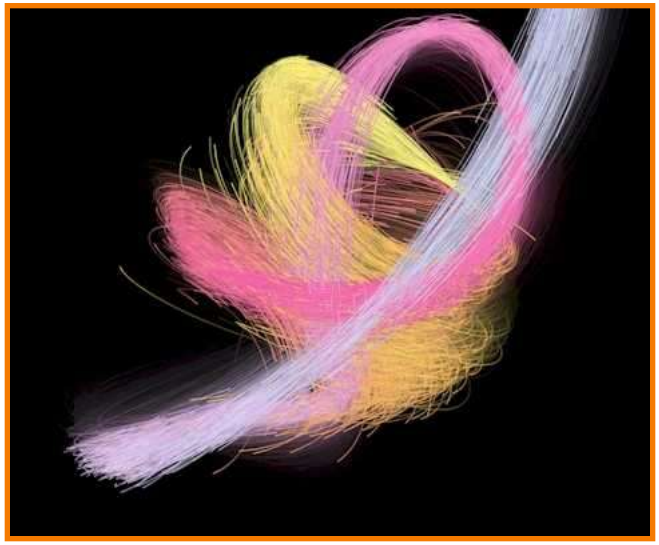
- Rendering styles
  - Points
  - Polygons
  - ➢ Shapes
  - Trails
  - etc.

McAllister

Rendering Particles

- Rendering styles
  - Points
  - Polygons
  - Shapes
  - ➢ Trails
  - etc.

McAllister



Particle Systems

- For each frame:
  - Create new particles and assign attributes
  - Delete any expired particles
  - ➢ Update particles based on attributes and physics
  - Render particles

McAllister

Equations of Motion

- Newton's Law for a point mass
  - f = ma

- Computing particle motion requires solving second-order differential equation

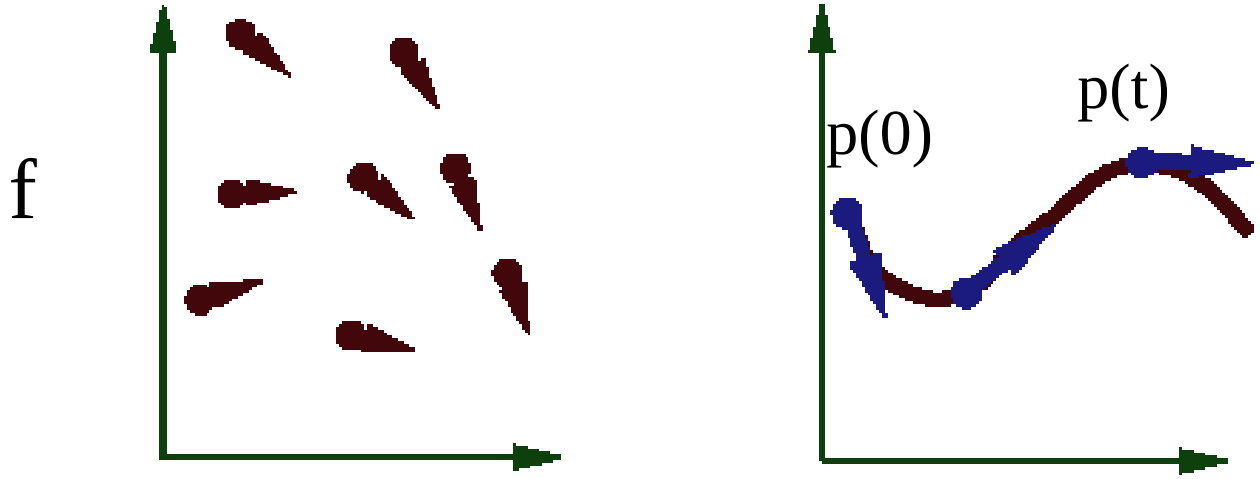
$$\ddot{x} = \frac{f(x, \dot{x}, t)}{m}$$

- Add variable v to form coupled first-order differential equations

$$\begin{cases} \dot{x} = v \\ \dot{v} = \dfrac{f}{m} \end{cases}$$

Solving the Equations of Motion

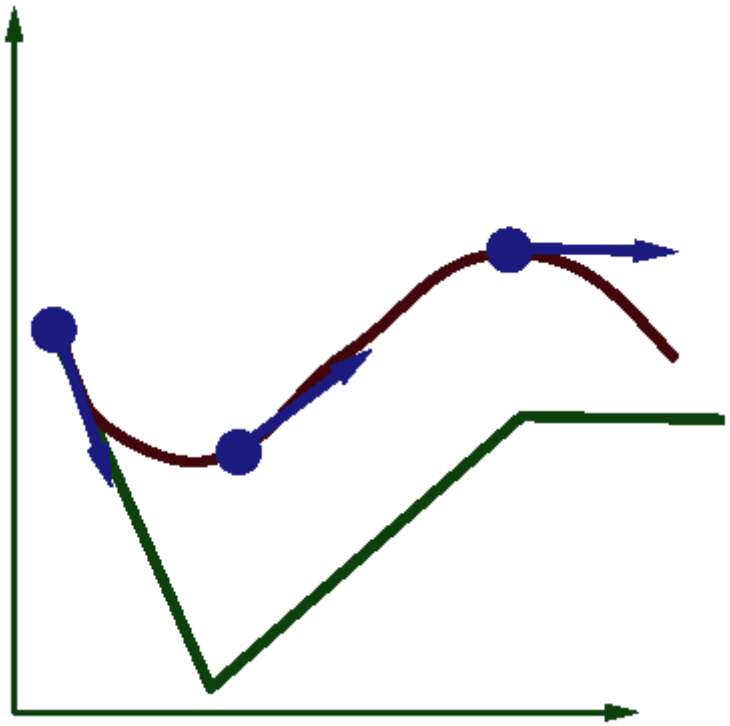
- Initial value problem
  - Know p(0), v(0), a(0)
  - Can compute force at any time and position
  - Compute p(t) by forward integration



Hodgins

Solving the Equations of Motion

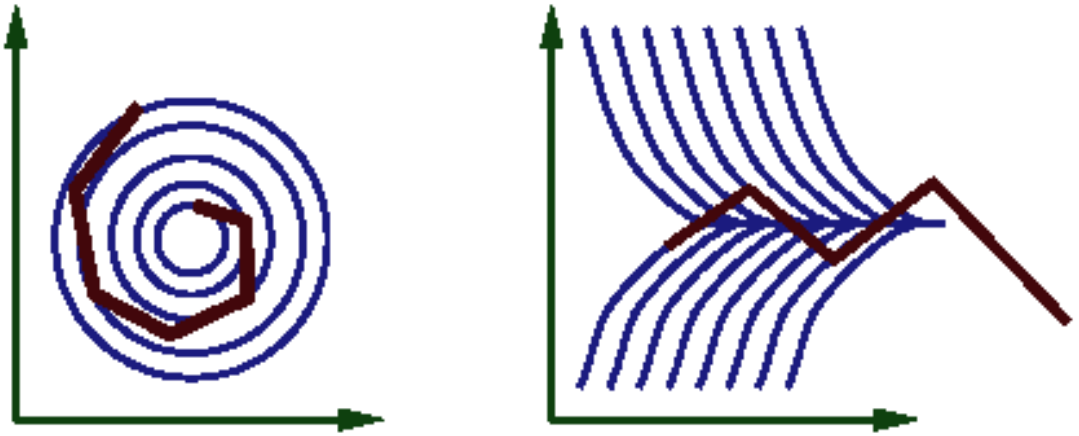
- Euler integration
  - $p(t+\otimes t)=p(t) + \otimes t\ v(t)$
  - $v(t+\otimes t)=v(t) + \otimes t\ f(x,t)/m$



Hodgins

Solving the Equations of Motion

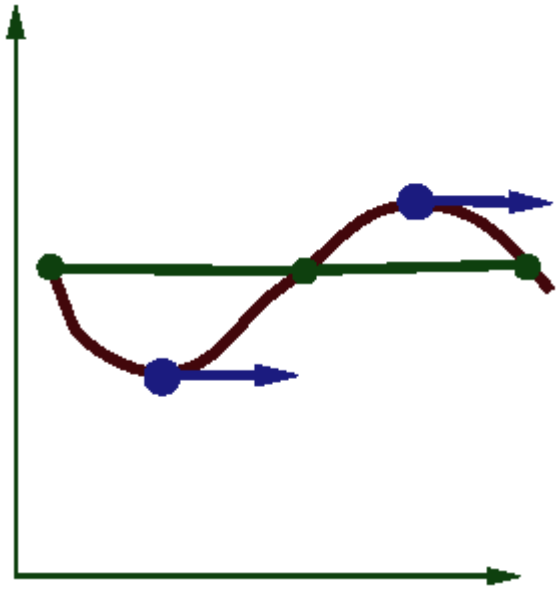
- Euler integration
  - $p(t+\otimes t)=p(t) + \otimes t\ v(t)$
  - $v(t+\otimes t)=v(t) + \otimes t\ f(p(t),t)/m$
- Problem:
  - Accuracy decreases as $\otimes t$ gets bigger



Hodgins

Solving the Equations of Motion

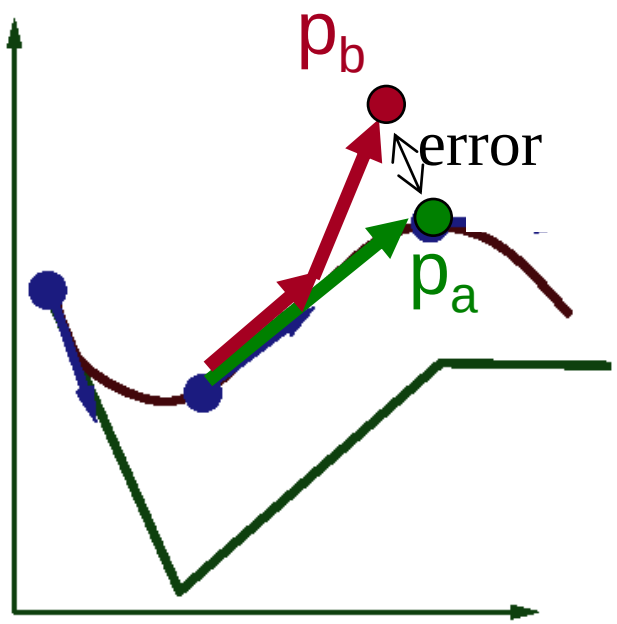
- Midpoint method (2nd order Runge-Kutta)
  - Compute an Euler step
  - Evalute f at the midpoint
  - Take an Euler step using midpoint force
    - » v(t+⊗t)=v(t) + ⊗t f(p(t) + 0.5*⊗t v(t),t)



Hodgins

Solving the Equations of Motion

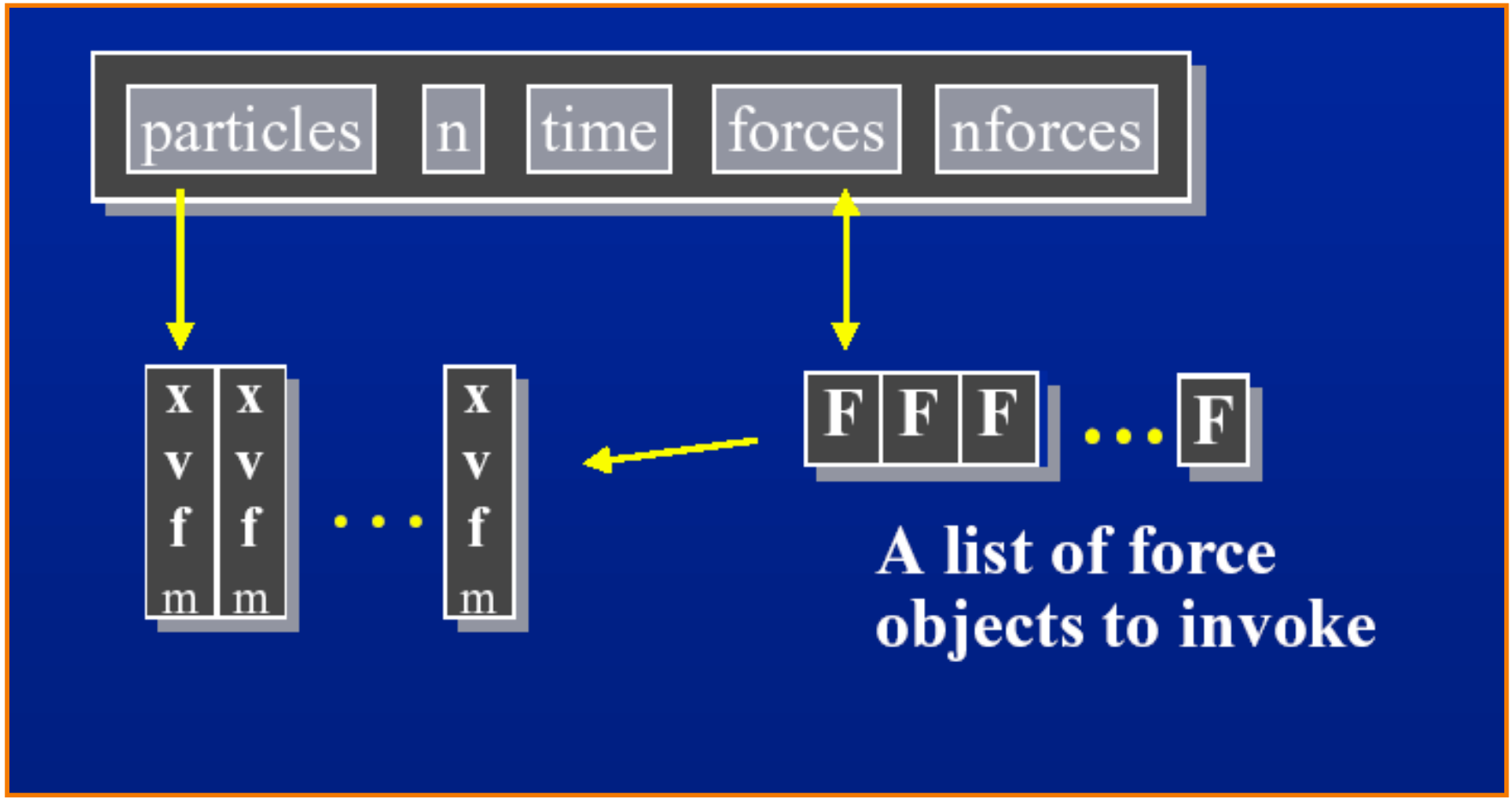
- Adapting step size
  - Compute p_a by taking one step of size h
  - Compute p_b by taking 2 steps of size h/2
  - Error = $| p_a - p_b |$
  - Multiply step size by factor (constant/error)



Particle System Forces

- Force fields
  - Gravity, wind, pressure
- Viscosity/damping
  - Liquids, drag
- Collisions
  - Environment
  - Other particles
- Other particles
  - Springs between neighboring particles (mesh)
  - Useful for cloth

Particle System Forces

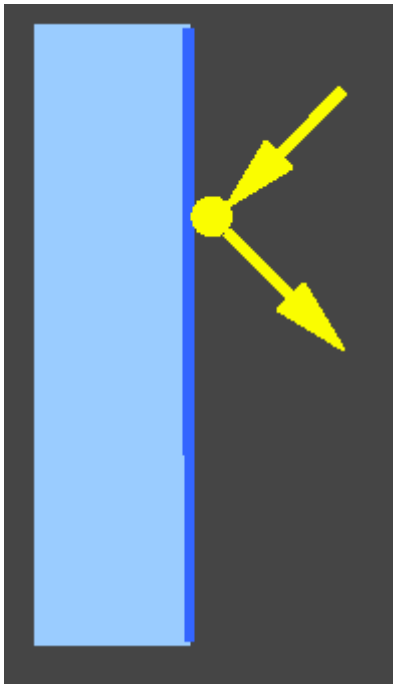


Example: Gravity

Example: Fire

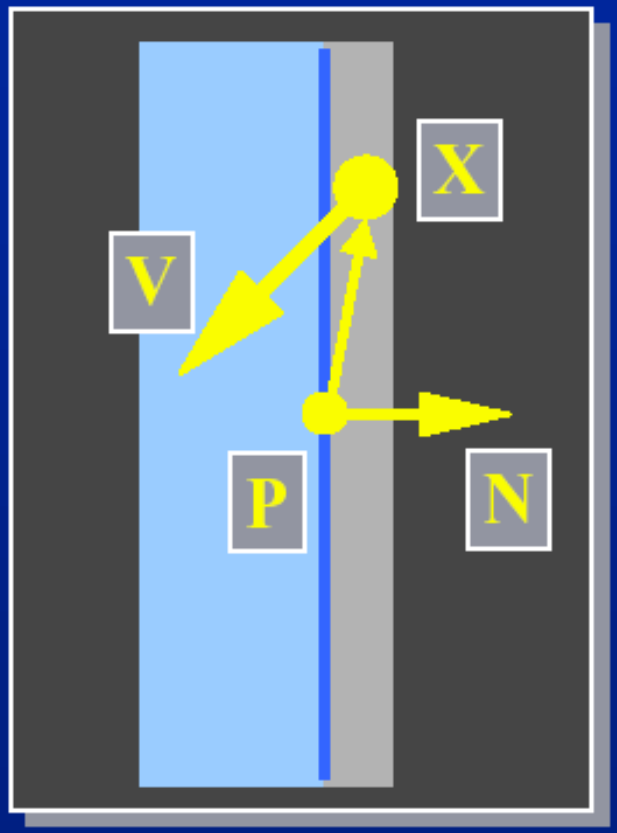
Example: Bouncing Off Wall

- Requires
  - Collision detection
  - Collision response (dynamic forces)



Example: Bouncing Off Wall

Collision Detection



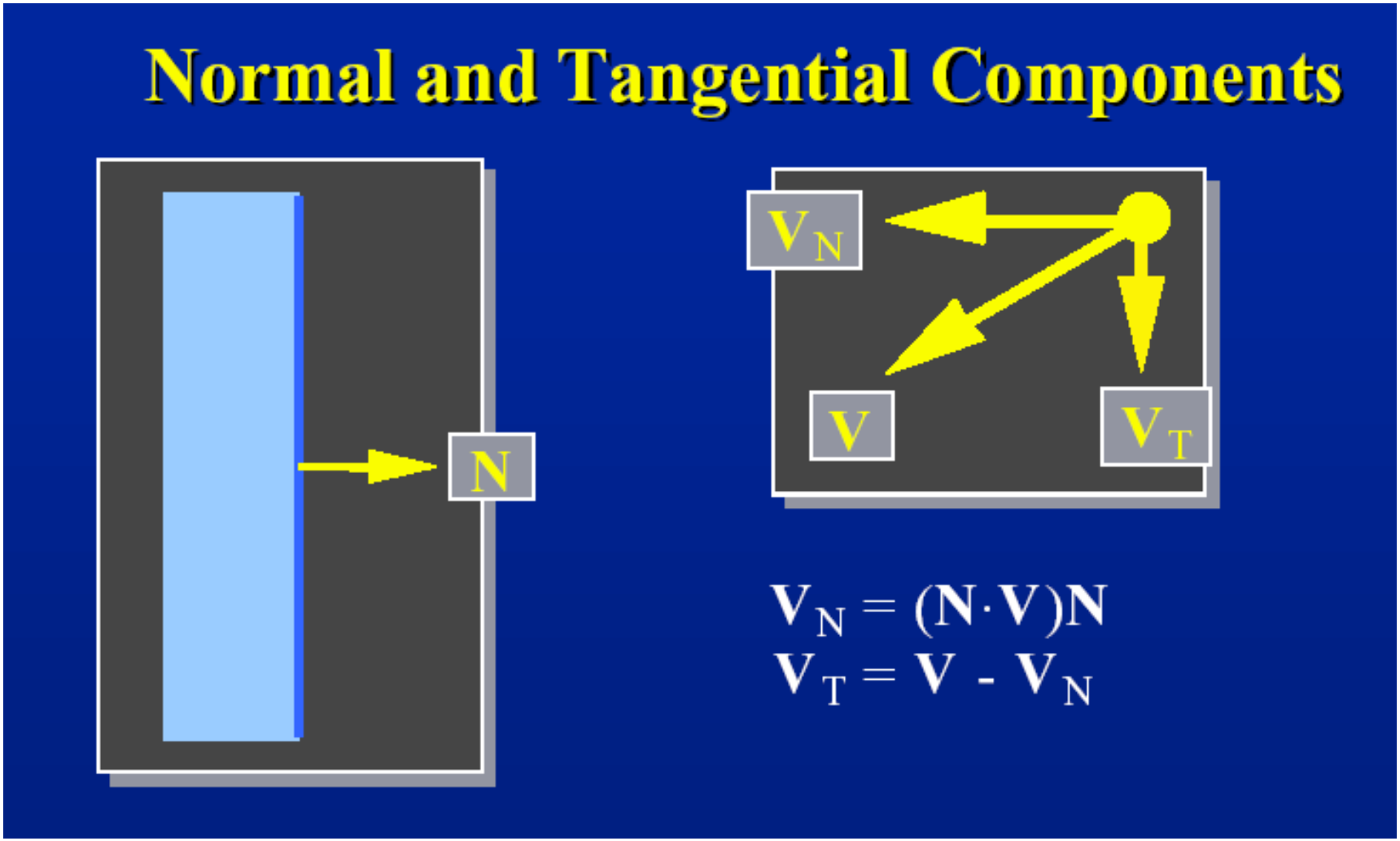
$$(X - P) \cdot N < \varepsilon$$

$$N \cdot V < 0$$

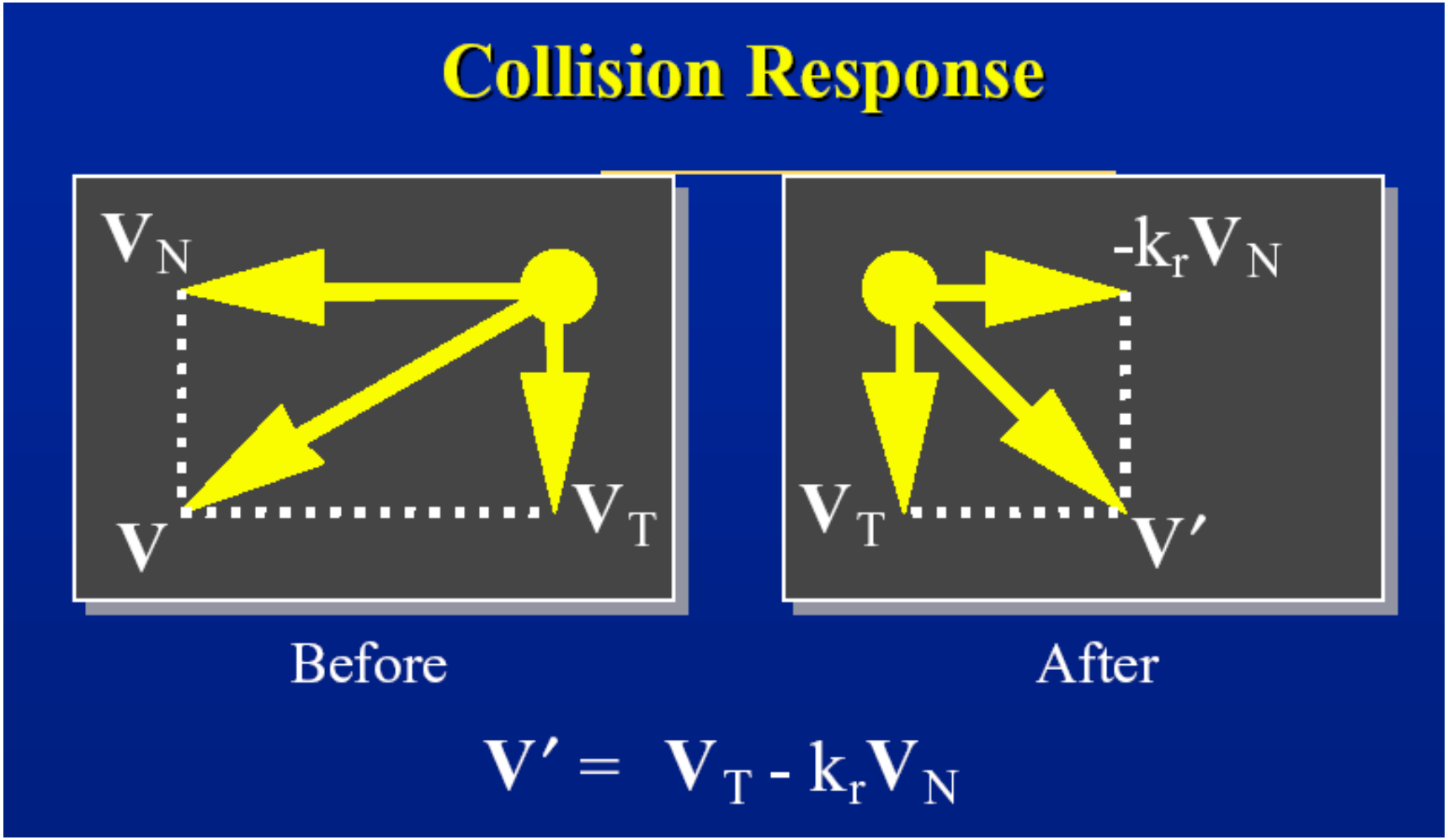
- Within ε of the wall.
- Heading in.

Witkin

Example: Bouncing Off Wall

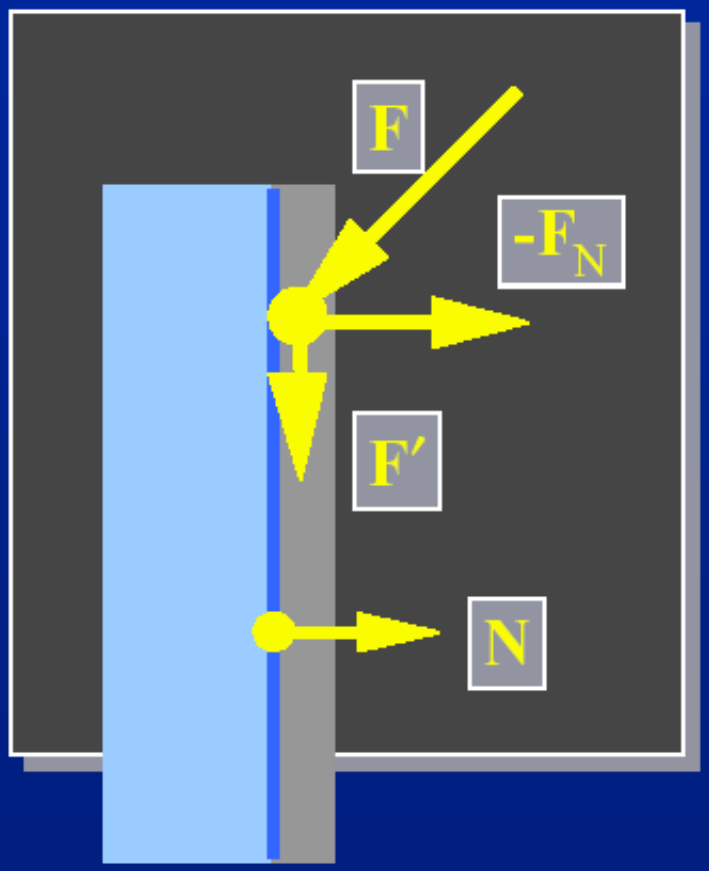


Example: Bouncing Off Wall



Witkin

Example: Bouncing Off Wall



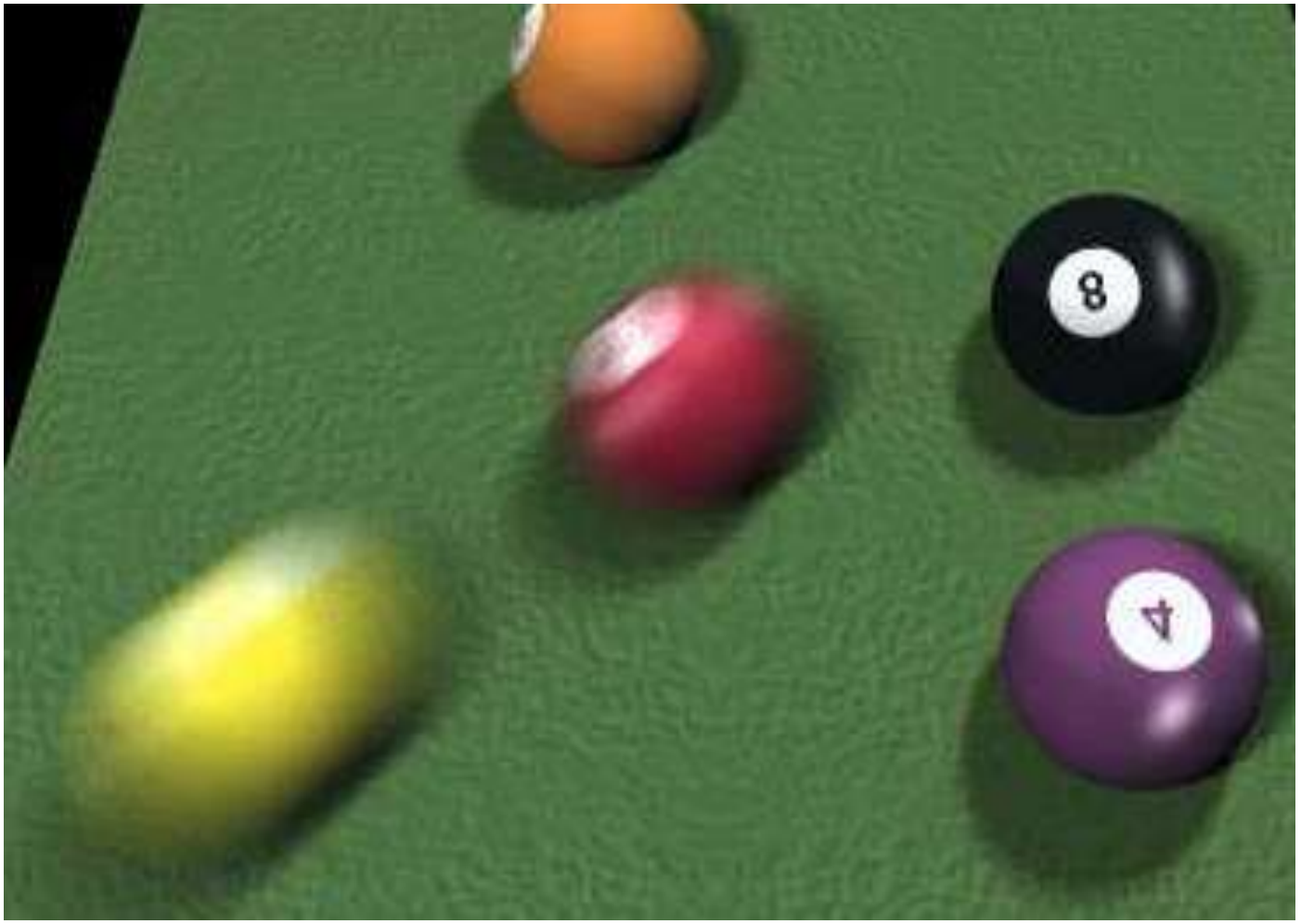
Contact Force

$\mathbf{F}' = \mathbf{F}_T$

The wall pushes back, cancelling the normal component of F.

(An example of a *constraint force.*)

Example: Bouncing Off Particles



Advancing our Particles

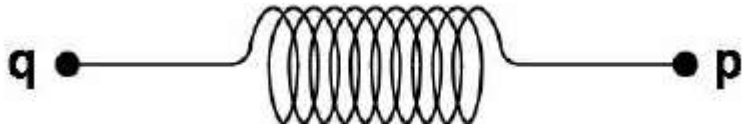
```
void Run(){
    while(!kbhit()){
       delay(10);
       cleardevice();
       //delay(10);
       for(int i=0;i<ParticleNum;i++){
         if(Particles[i].age >= 0){
          setfillstyle(1,Particles[i].color);
          setcolor(Particles[i].color);
          //circle((int)Particles[i].Pos.x,(int)Particles[i].Pos.y,Particles[i].size);
          fillellipse(…);
          Particles[i].Vel.x+=TotForce.x;
          Particles[i].Vel.y+=TotForce.y;
          Particles[i].Pos.x+=Particles[i].Vel.x;
          Particles[i].Pos.y+=Particles[i].Vel.y;
          Particles[i].age++;
         if(Particles[i].age > Particles[i].LifeSpan){
         if(Particle_Type == Create) {
          InitParticle(i);
         }
         else {
          Particles[i].age = -1;
         } }

    }
    }
    } }
```

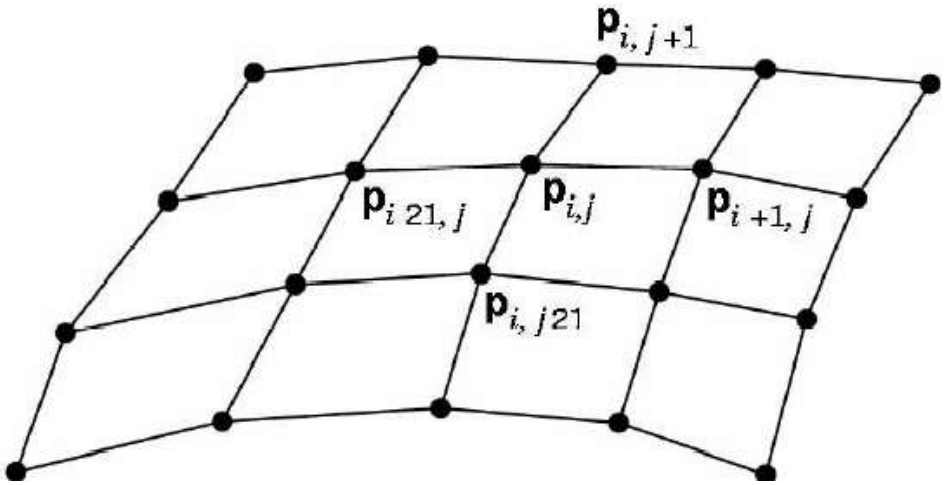
Example: Cloth

- Spring-mass mesh
- Hooke's law

$$\mathbf{f} = -k_s(|\mathbf{d}| - s)\frac{\mathbf{d}}{|\mathbf{d}|}$$

f = force
k_s = spring constant
d = p - q
s = resting length





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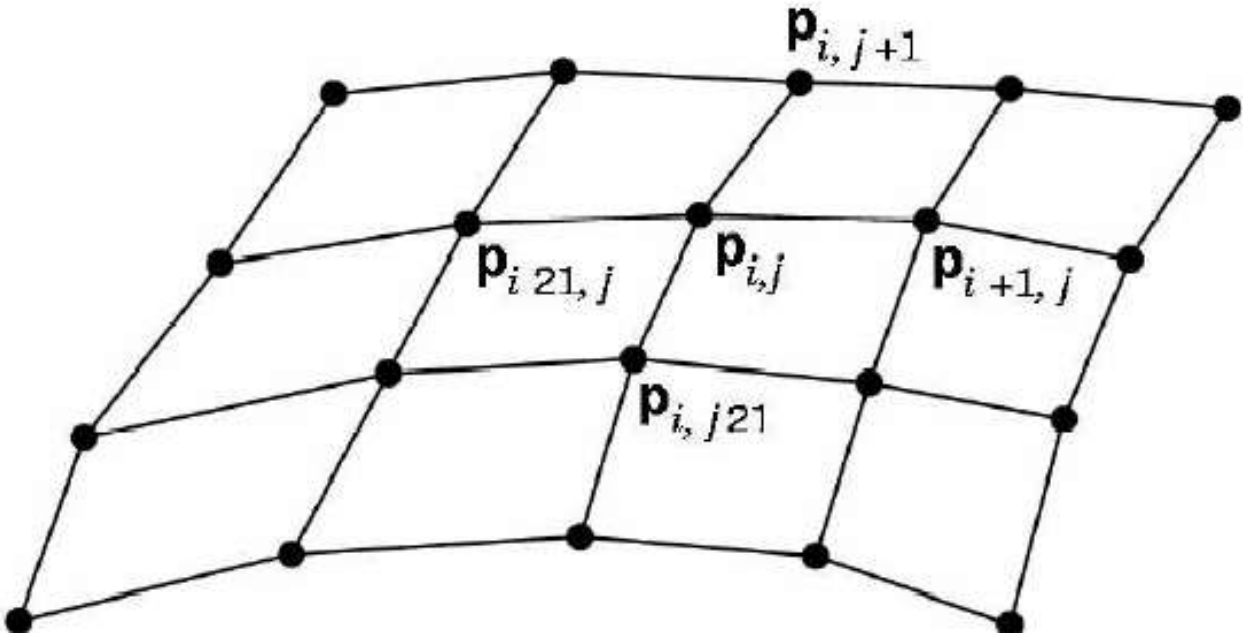
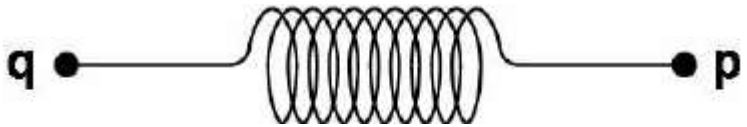
Example: Cloth

- Spring-mass mesh





Hodgins

Example: Cloth

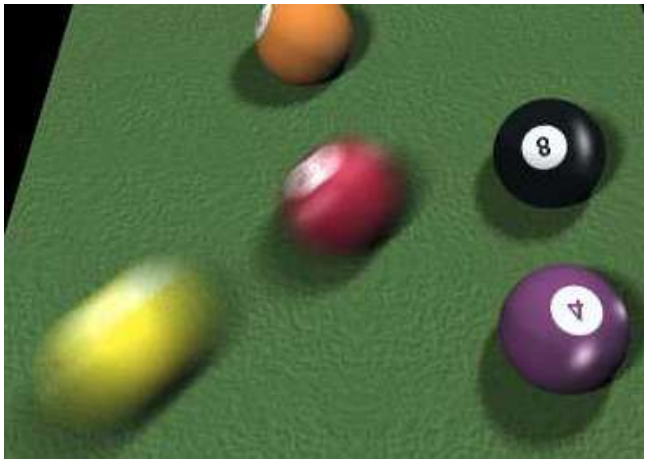
Animating Developable Surfaces

Simulating Knitted Cloth at the Yarn Level

Example: Flocks & Herds

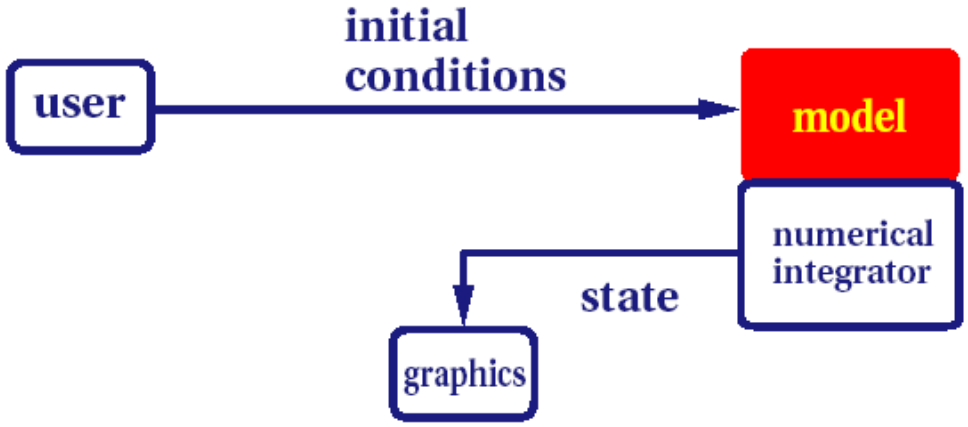
Summary

- Particle systems
  - Lots of particles
  - Simple physics
- Interesting behaviors
  - Waterfalls
  - Smoke
  - Cloth
  - Flocks
- Solving motion equations
  - Simplest method is Euler integration
  - Better to use adaptive step sizes



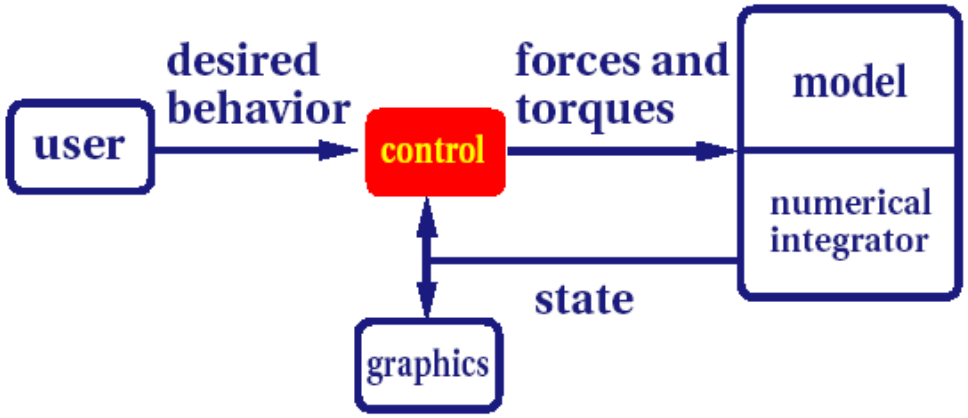
Passive vs. Active Dynamics

Passive--no muscles or motors



particle systems
leaves
water spray
clothing

Active--internal source of energy

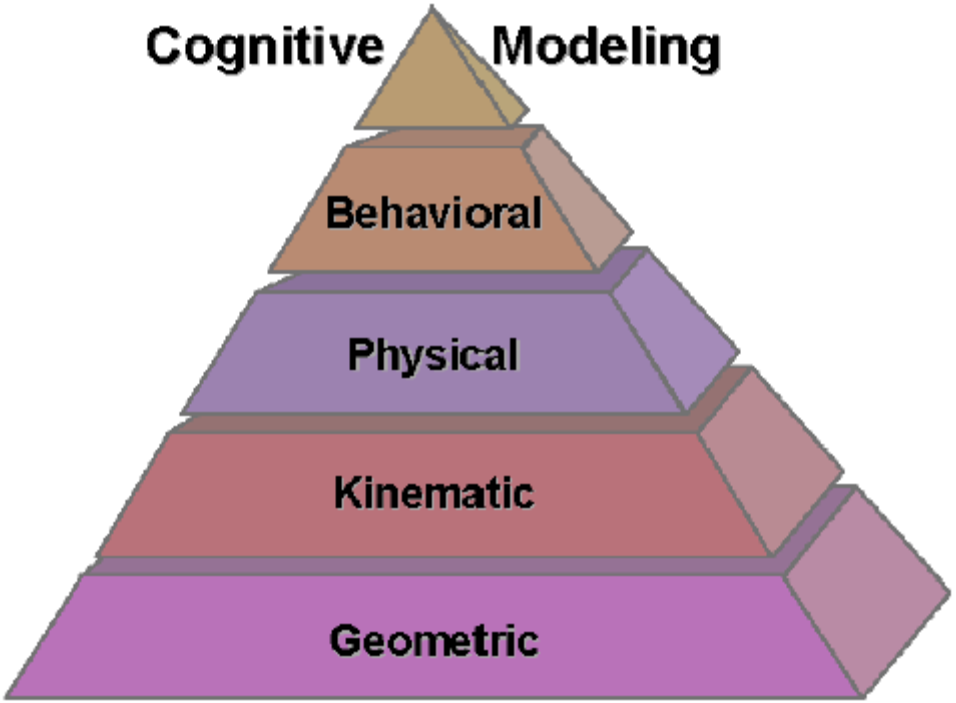


running human
trotting dog
swimming fish

Hodgins

Active Dynamics

- Motions
  - Physics
  - Controllers
  - Learning
- Behaviors
  - States
- Cognition
  - Planning



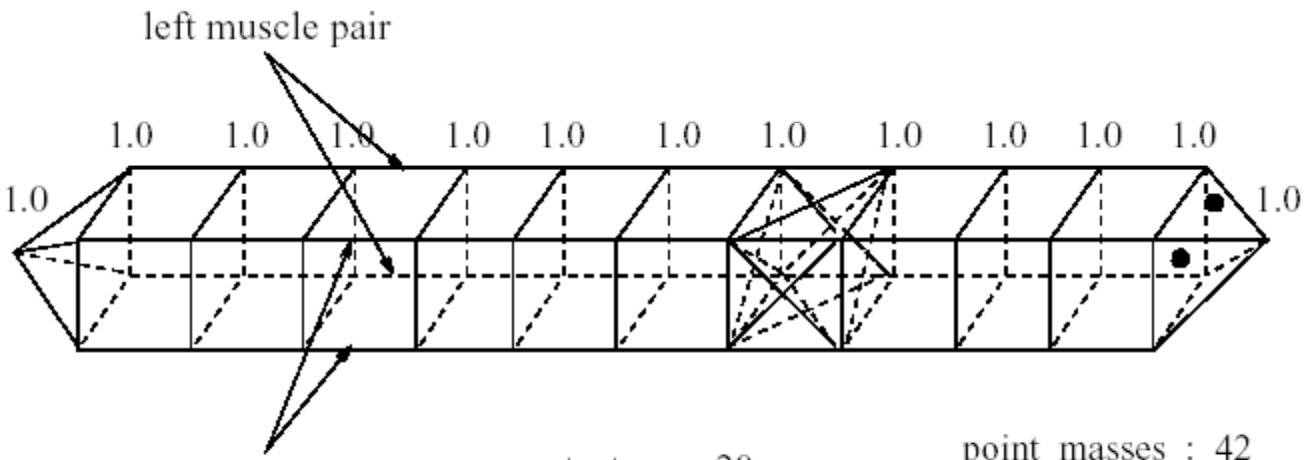
Funge99

Motion

- Example 1: how do worms move?

Snake Motion

Worm Biomechanical Model



actuators : 20

springs' stiffness : 50.0

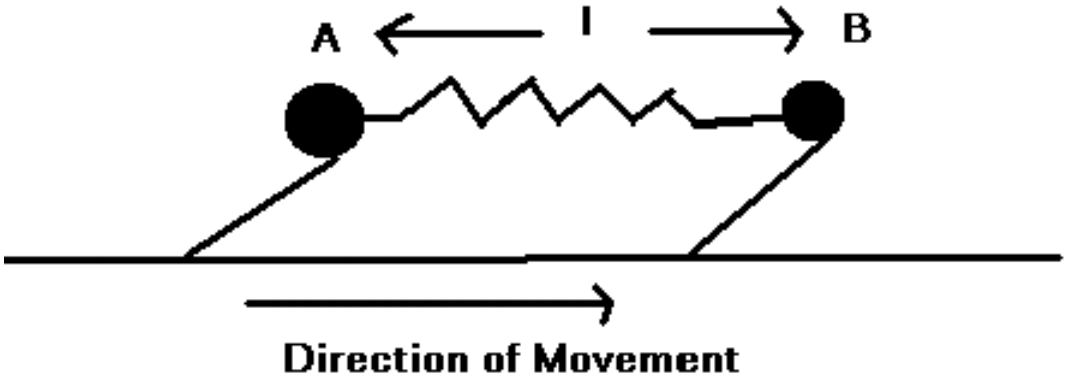
point masses : 42

DOFs : 126

size of the state space : 252

Grzeszczuk95

Worm Physics



$$f = k(L - I) - D\frac{dl}{dt}$$

$$a = f / m$$

$$X = \frac{1}{m}\int\int f dt dt$$

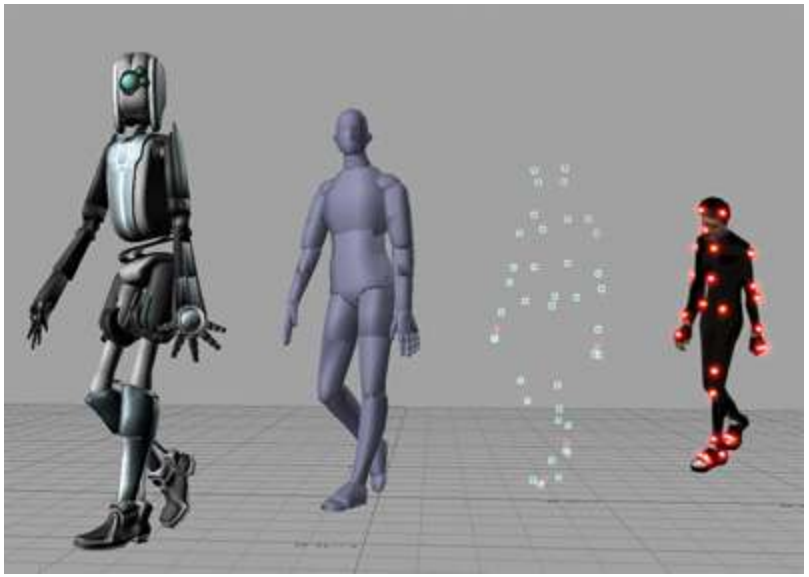
f = force along spring direction
k = spring force constant
D = damping force
I = current spring length
L = minimum energy spring length

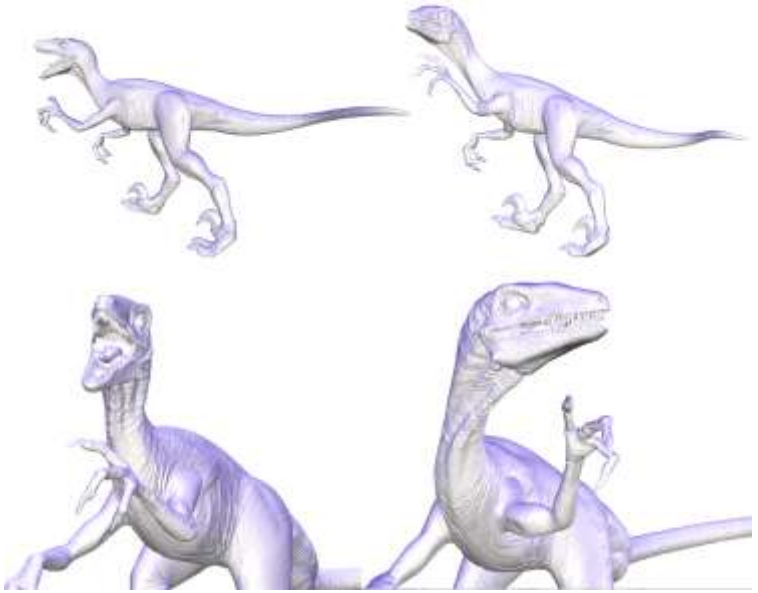
... plus forces due to friction with ground.

Miller88

Her Majesty's Secret Serpent

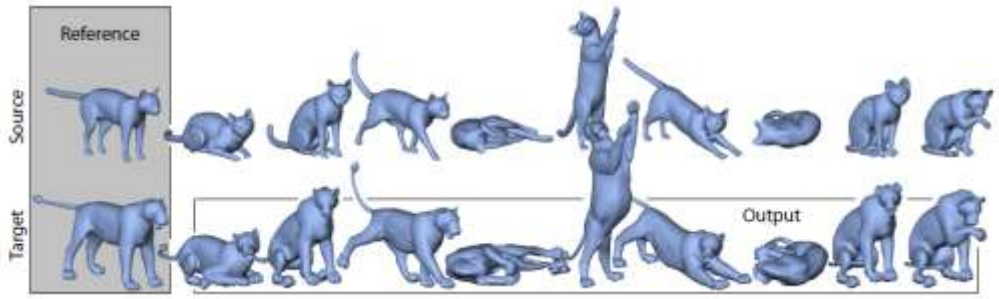
Miller89





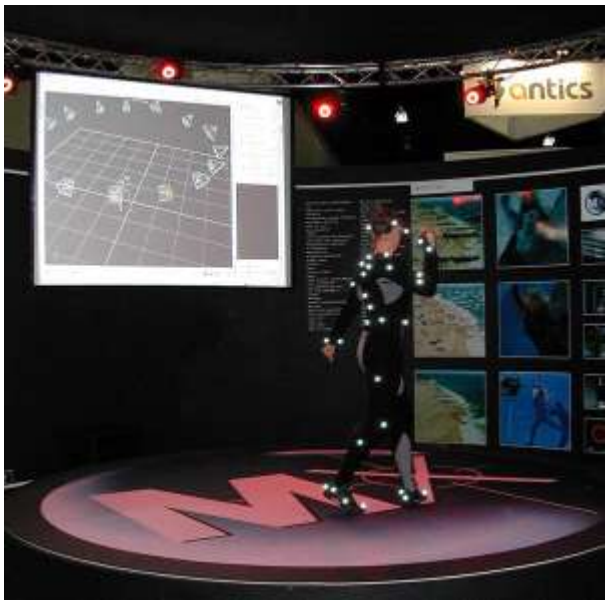
Time permitting…

OTHER TOPICS

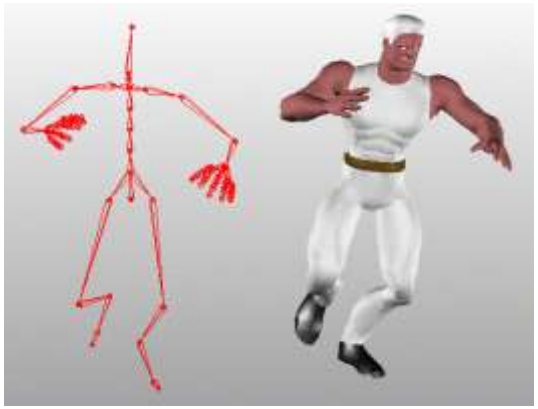


Other topics in Animation

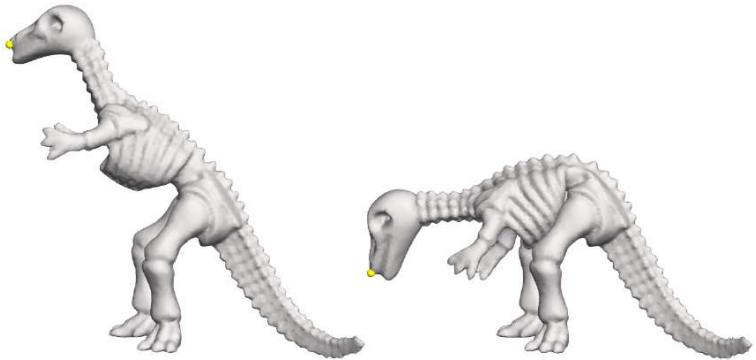
- Motion Capture



- Skinning

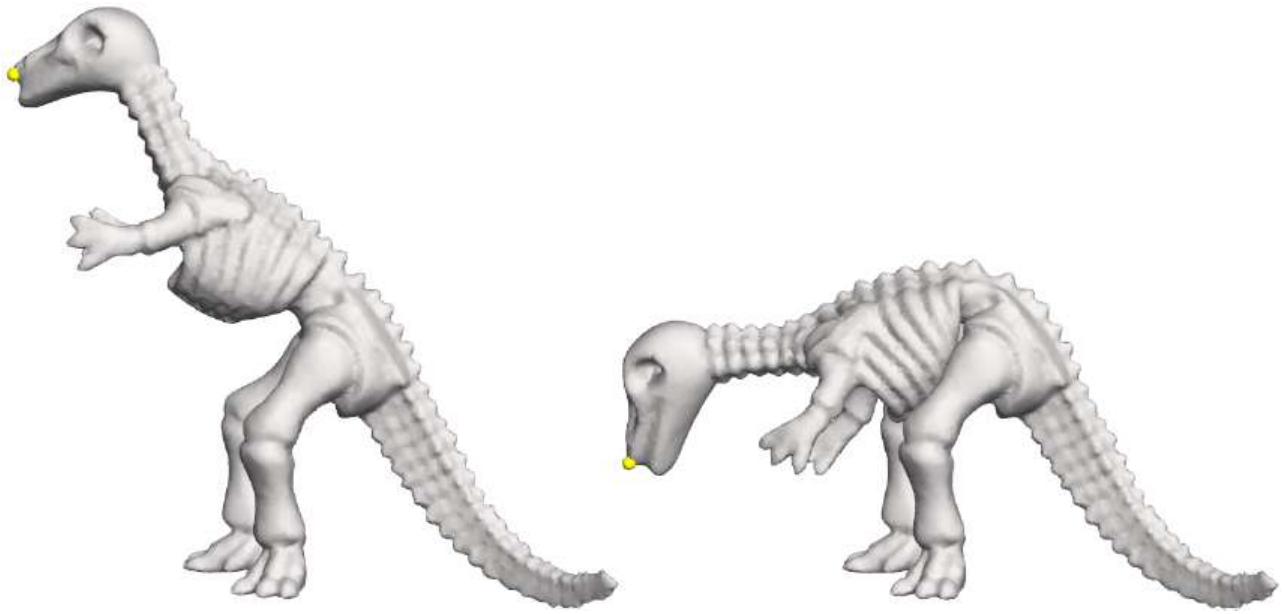


- Deformations
  - Differential representation
  - Cage deformations
  - Deformation transfer



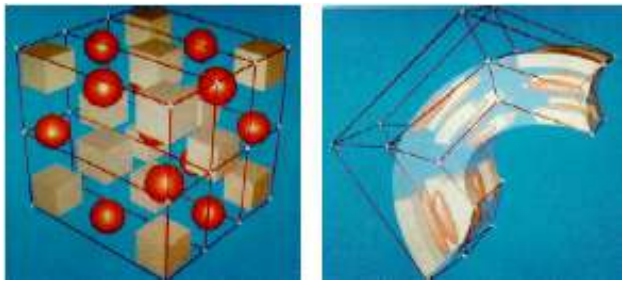
What do we expect from surface deformation?

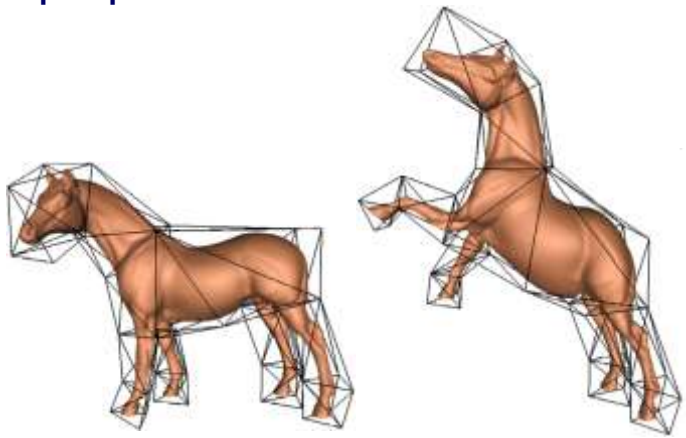
- Smooth effect on the large scale
- As-rigid-as-possible effect on the small scale (preserves details)



Several approaches

- FFD (space deformation)
  - Lattice-based (Sederberg & Parry 86, Coquillart 90, …)
  - Curve-/handle-based (Singh & Fiume 98, Botsch et al. 05, …)
  - Cage-based (Ju et al. 05, Joshi et al. 07, Kopf et al. 07)
- Pros:
  - efficiency almost independent of the surface resolution
  - possible reuse
- Cons:
  - space warp, so can't precisely control surface properties

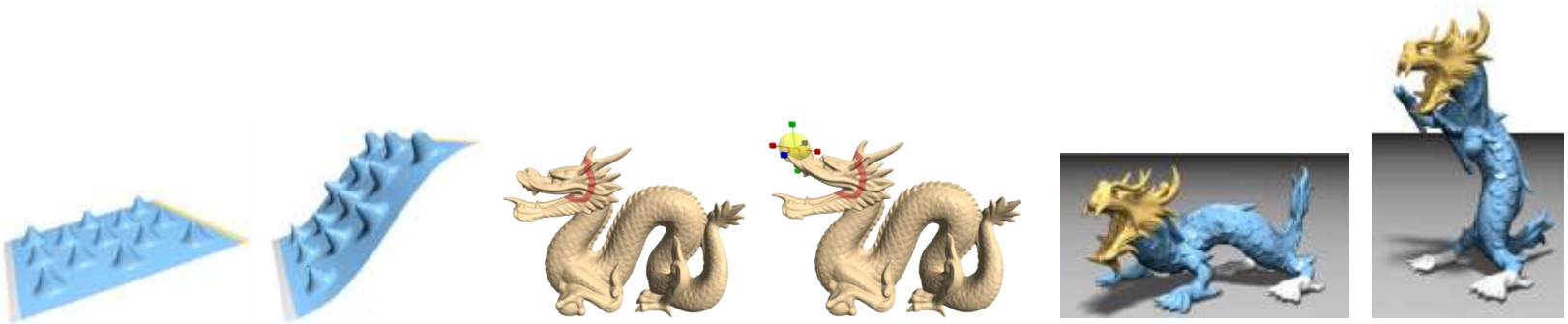




images taken from [Sederberg and Parry 86] and [Ju et al. 05]

Several approaches

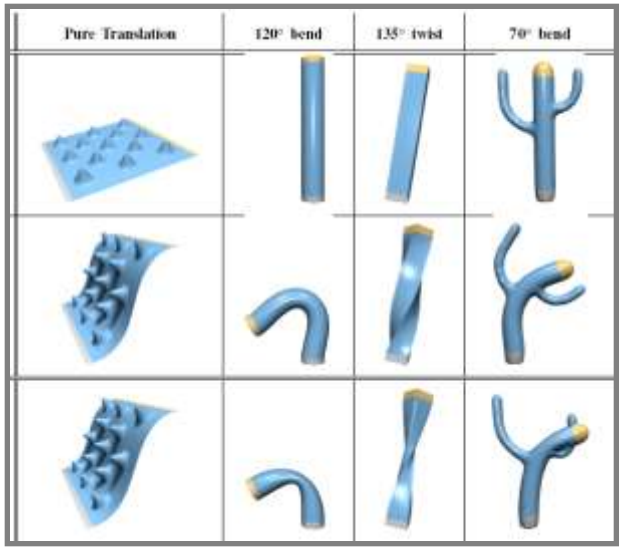
- Surface-based approaches
  - Multiresolution modeling
    Zorin et al. 97, Kobbelt et al. 98, Lee 98, Guskov et al. 99, Botsch and Kobbelt 04, …
  - Differential coordinates – linear optimization
    Lipman et al. 04, Sorkine et al. 04, Yu et al. 04, Lipman et al. 05, Zayer et al. 05, Botsch et al. 06, Fu et al. 06, …
  - Non-linear global optimization approaches
    Kraevoy & Sheffer 04, Sumner et al. 05, Hunag et al. 06, Au et al. 06, Botsch et al. 06, Shi et al. 07, …

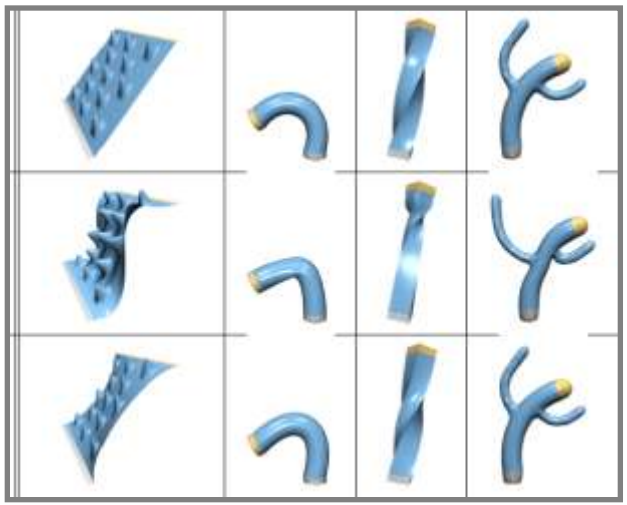


images taken from PriMo, Botsch et al. 06

Surface-based approaches

- Pros:
  - direct interaction with the surface
  - control over surface properties
- Cons:
  - linear optimization suffers from artifacts (e.g. translation insensitivity)
  - non-linear optimization is more expensive and non-trivial to implement





Sorkine et al 2004

LAPLACIAN SURFACE EDITING

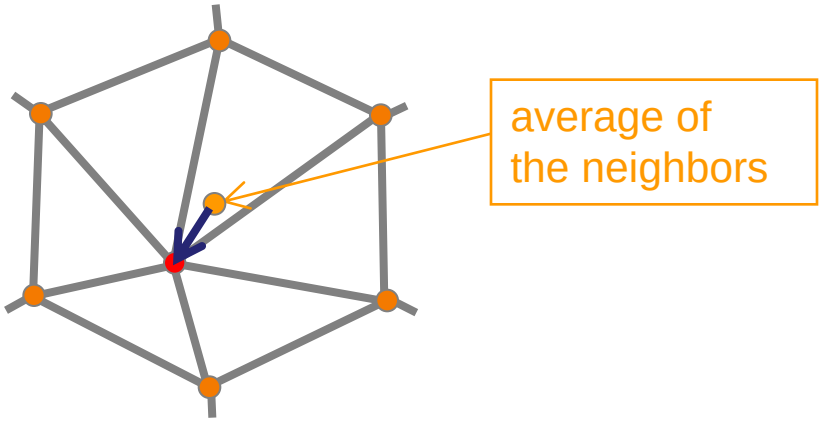
http://www.cs.nyu.edu/~sorkine/ProjectPages/Editing/lse.html

Differential Coordinates

- **Differential coordinates** are defined by the discrete Laplacian operator:

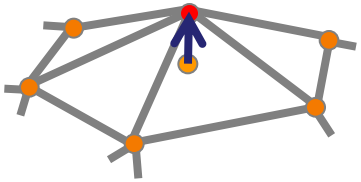
$$\delta_i = L(\mathbf{v}_i) = \mathbf{v}_i - \frac{1}{d_i} \sum_{j \in N(i)} \mathbf{v}_j$$

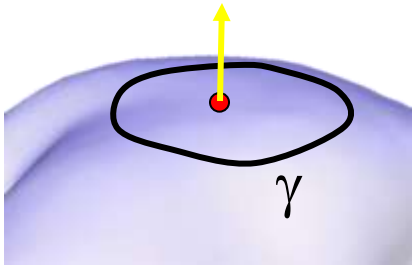
- For highly irregular meshes: cotangent weights [Desbrun et al. 99]



Why differential coordinates?

- They represent the **local** detail / local shape description
  - The direction approximates the normal
  - The size approximates the mean curvature





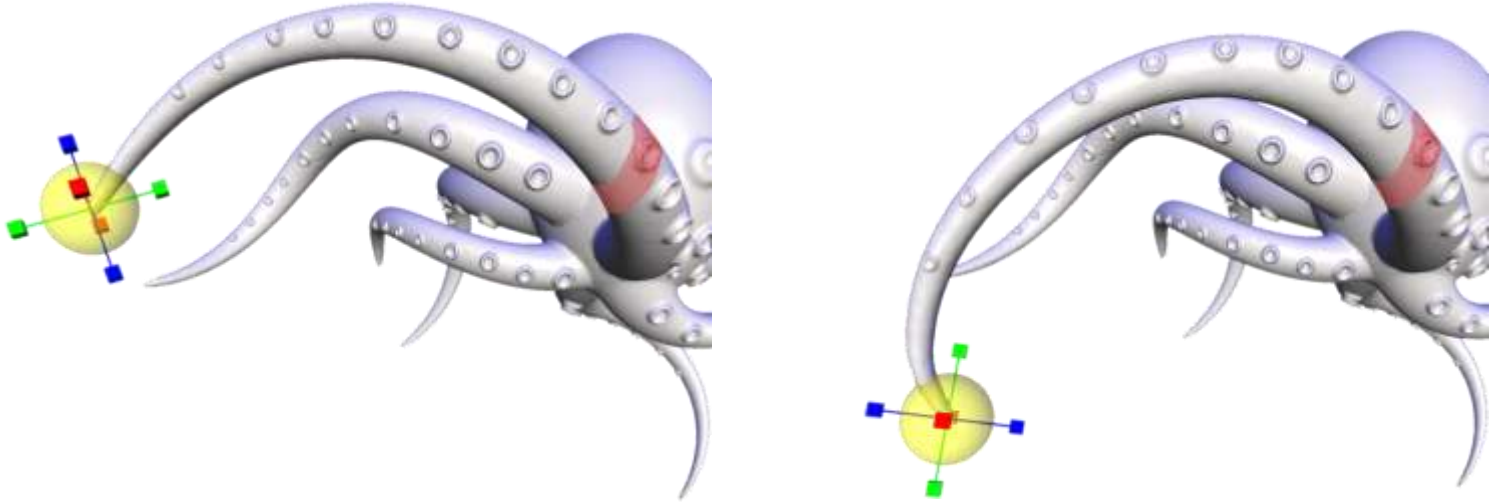
$$\boldsymbol{\delta}_i = \frac{1}{d_i} \sum_{\mathbf{v} \in N(i)} (\mathbf{v}_i - \mathbf{v})$$

$$\frac{1}{len(\gamma)} \int_{\mathbf{v} \in \gamma} (\mathbf{v}_i - \mathbf{v}) ds$$

$$\lim_{len(\gamma) \to 0} \frac{1}{len(\gamma)} \int_{\mathbf{v} \in \gamma} (\mathbf{v}_i - \mathbf{v}) ds = H(\mathbf{v}_i)\mathbf{n}_i$$

Why differential coordinates?

- Local detail representation – enables detail preservation through various modeling tasks
- Representation with sparse matrices
- Efficient linear surface reconstruction



Overall framework

- Compute differential representation

$$\Delta = L(V)$$

- Pose modeling constraints

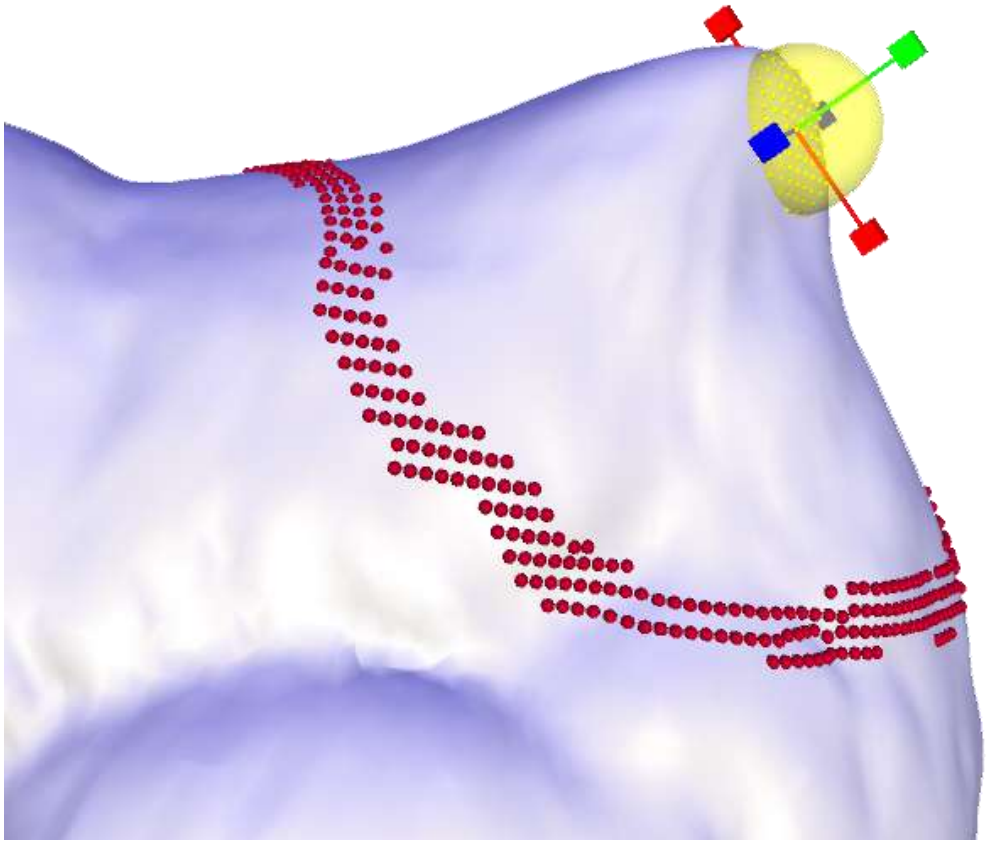
$$\mathbf{v}'_i = \mathbf{u}_i, \quad i \in C$$

- Reconstruct the surface – in least-squares sense

$$\tilde{V}' = \arg\min_{V'} \left(\|L(V') - \Delta\|^2 + \sum_{i \in C} \|\mathbf{v}'_i - \mathbf{u}_i\|^2 \right)$$

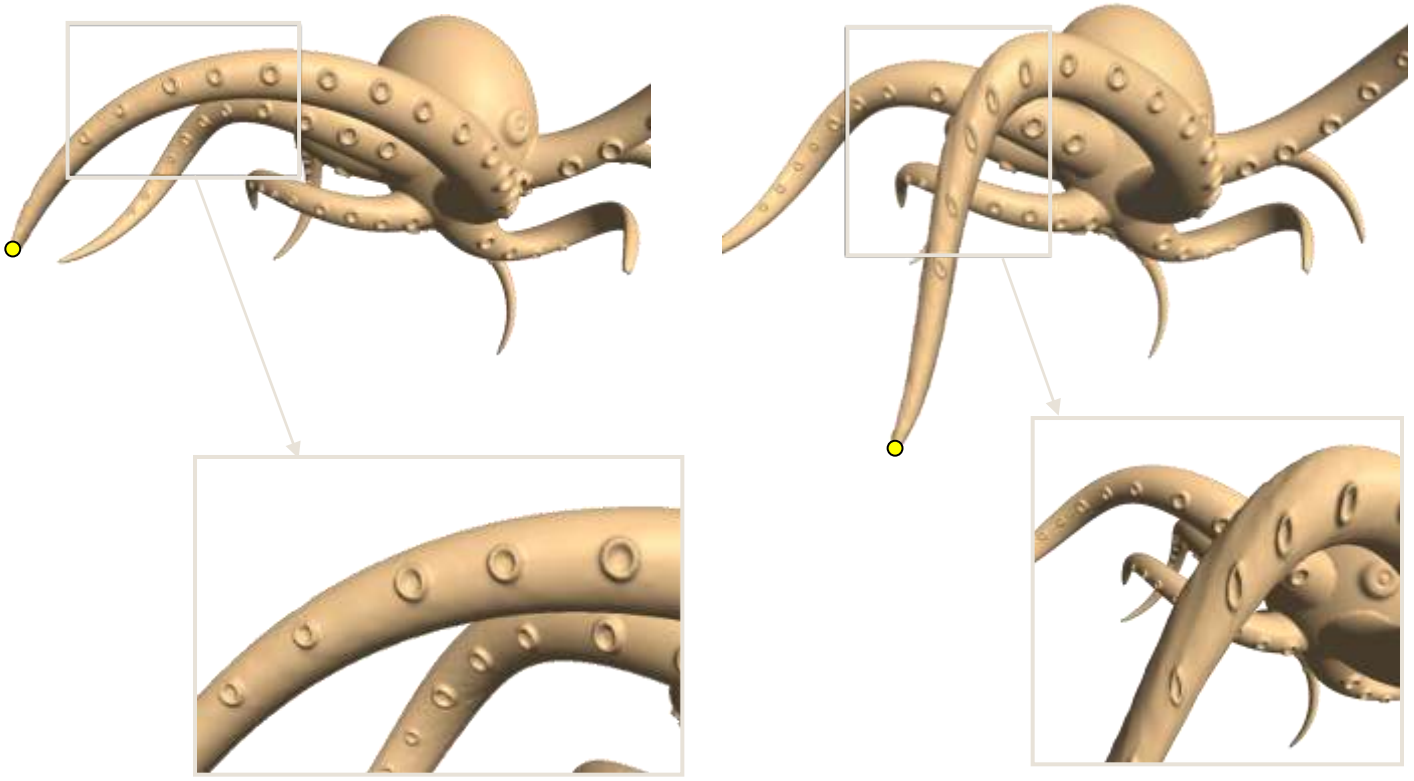
Overall framework

- ROI is bounded by a belt (static anchors)
- Manipulation through handle(s)



Problem: invariance to transformations

- The basic Laplacian operator is translation-invariant, but not rotation- and scale-invariant
- Reconstruction attempts to preserve the original global orientation of the details



Implicit definition of transformations

- The idea: solve for local transformations AND the edited surface simultaneously!

$$\tilde{V}' = \arg\min_{V'} \left(\sum_{i=1}^{n} \left\| L(\mathbf{v}'_i) - T_i(\boldsymbol{\delta}_i) \right\|^2 + \sum_{j \in C} \left\| \mathbf{v}'_j - \mathbf{u}_j \right\|^2 \right)$$

Transformation
of the local frame

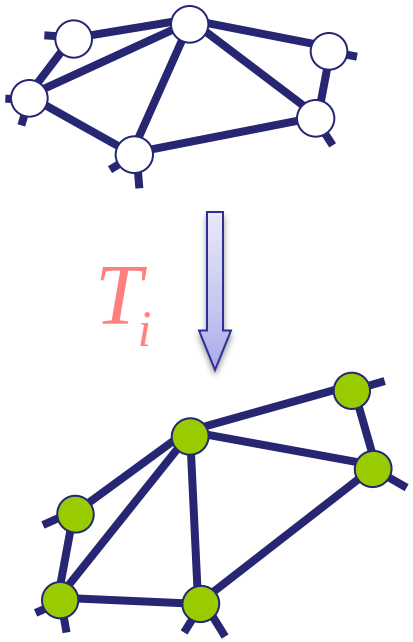
Defining the transformations T_i

$$\tilde{V}' = \arg\min_{V'} \left(\sum_{i=1}^{n} \left\| L(\mathbf{v}'_i) - T_i(\boldsymbol{\delta}_i) \right\|^2 + \sum_{j \in C} \left\| \mathbf{v}'_j - \mathbf{u}_j \right\|^2 \right)$$

- How to formulate T_i?
  - Based on the local (1-ring) neighborhood
  - Linear dependence on the unknown $\mathbf{v}_i$'

Members of the 1-ring of i-th vertex

$$\begin{aligned} \mathbf{v}'_{i_1} &= T_i \mathbf{v}_{i_1} \\ \mathbf{v}'_{i_2} &= T_i \mathbf{v}_{i_2} \\ &\vdots \\ \mathbf{v}'_{i_k} &= T_i \mathbf{v}_{i_k} \end{aligned}$$



Defining the transformations T_i

- First attempt: define T_i simply by solving

$$T_i = \arg\min_{T_i} \sum_{j=1}^{k} \left\| \mathbf{v}'_{i_j} - T_i \mathbf{v}_{i_j} \right\|^2$$

$$\begin{pmatrix} \\ T_i \\ \\ \end{pmatrix} = \begin{pmatrix} | & | & & | \\ \mathbf{v}'_{i_1} & \mathbf{v}'_{i_2} & \cdots & \mathbf{v}'_{i_k} \\ | & | & & | \end{pmatrix} \begin{pmatrix} | & | & & | \\ \mathbf{v}_{i_1} & \mathbf{v}_{i_2} & \cdots & \mathbf{v}_{i_k} \\ | & | & & | \end{pmatrix}^*$$

Defining the transformations T_i

- Plug the expressions for T_i into the least-squares reconstruction formula:

$$\tilde{V}' = \arg\min_{V'} \left(\sum_{i=1}^{n} \| L(\mathbf{v}'_i) - T_i \boldsymbol{\delta}_i \|^2 + \sum_{j \in C} \| \mathbf{v}'_j - \mathbf{u}_j \|^2 \right)$$

Linear combination of the unknown $\mathbf{v}_i'$

Constraining T_i

- Trivial solution for T_i will result in membrane surface reconstruction
- To preserve the shape of the details we constrain T_i to **rotations, uniform scales and translations**

$$T_i = \begin{pmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{pmatrix}$$

Linear constraints on t_{lm}
so that T_i is
rotation+scale+translation
??

Constraining T_i – 3D case

- Not linear in 3D:

$$\begin{pmatrix}\text{rotation} + \\ \text{uniform scale}\end{pmatrix} = s \exp H = s\left(\alpha I + \beta H + \mathbf{h}^T\mathbf{h}\right)$$

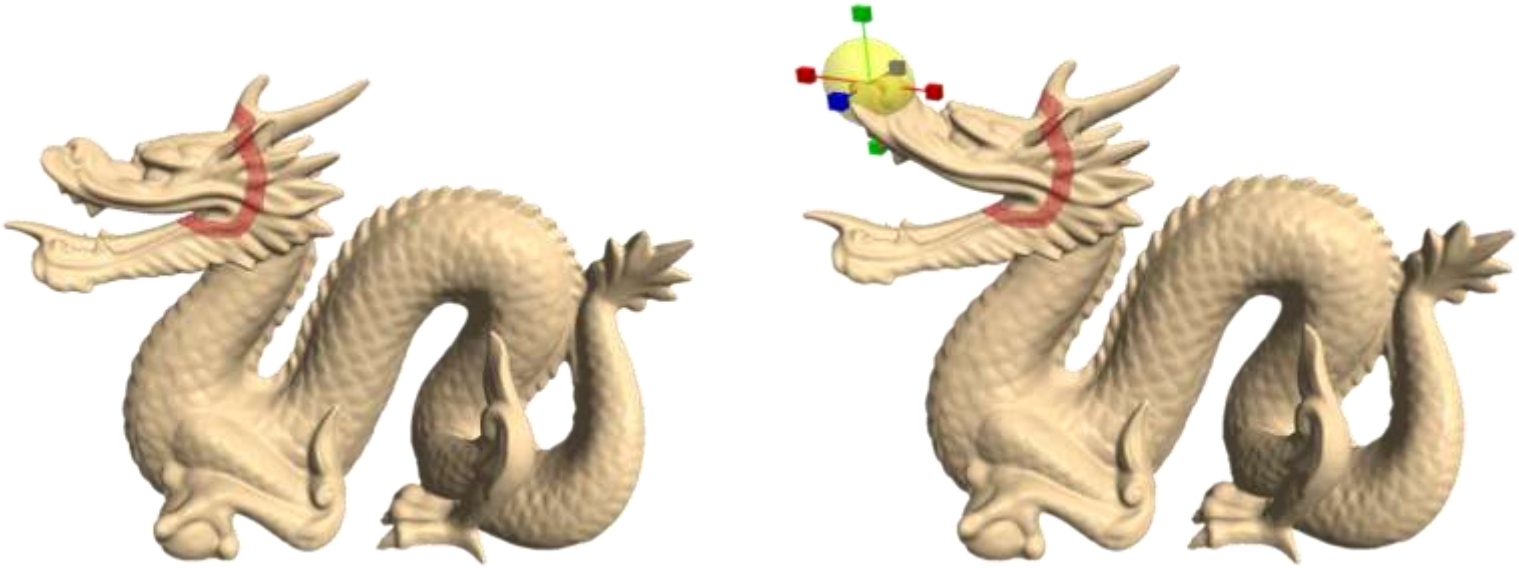
H is 3×3 skew-symmetric, $H\mathbf{x} = \mathbf{h}\times\mathbf{x}$

- Linearize by dropping the quadratic term

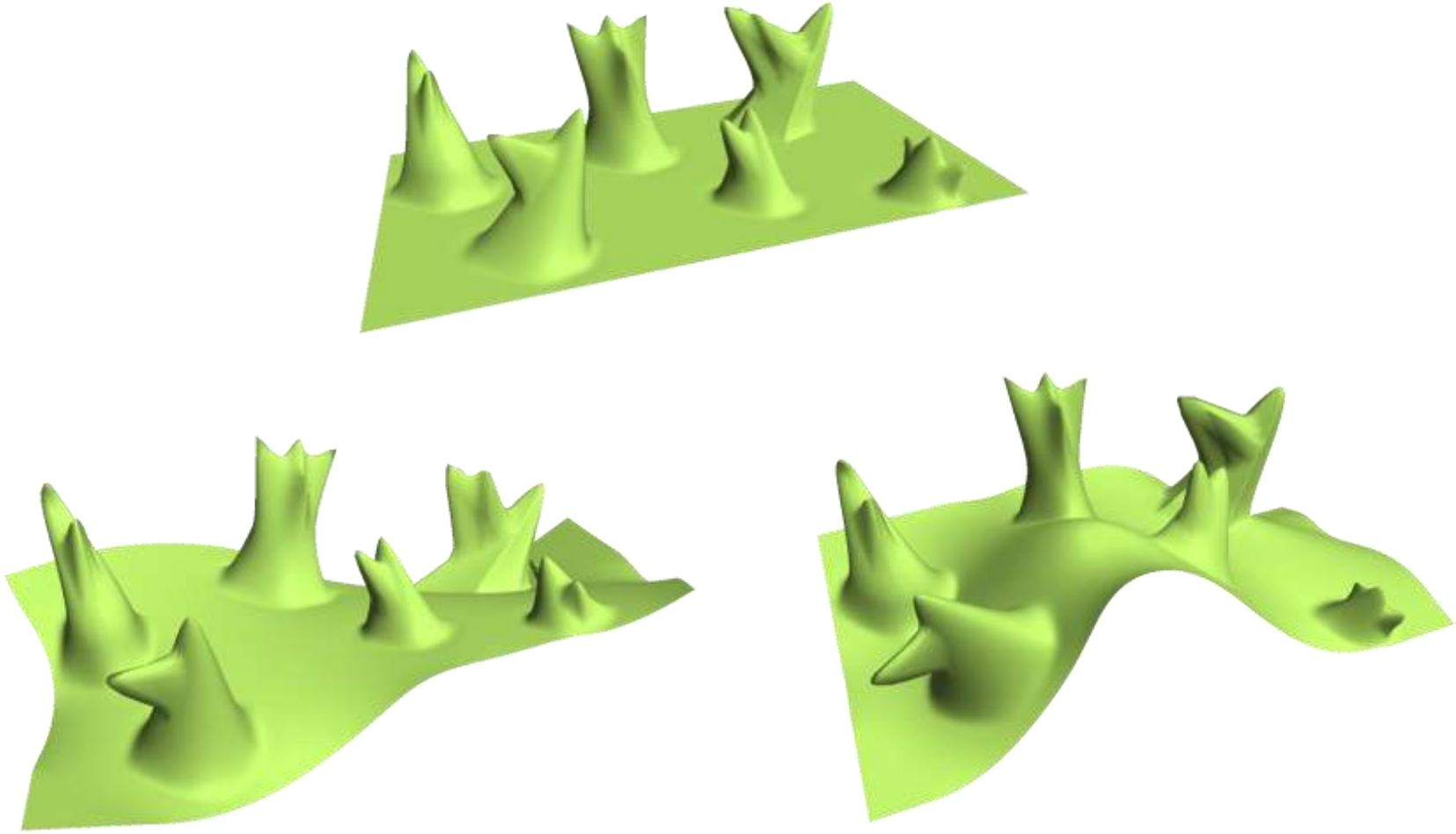
Adjusting T_i

- Due to linearization, T_i scale the space along the $\mathbf{h}$ axis by $cos\theta$
- When θ is large, this causes anisotropy
- Possible correction:
  - Compute T_i , remove the scaling component and reconstruct the surface again from the corrected δ_i
  - Apply our technique from [Lipman et al. 04] first, and then the current technique – with small θ.

Some results



Some results



Some results