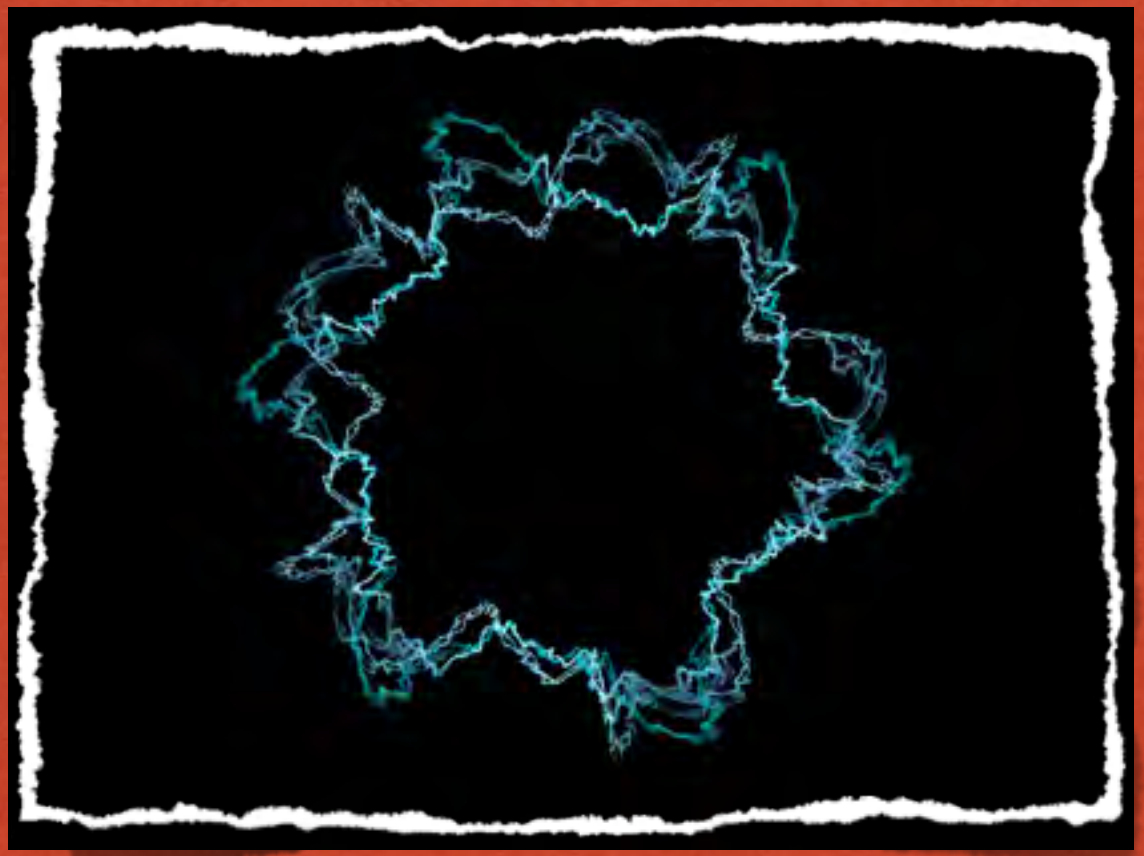
Turing 102

1912–1954

Systems

Levels of Abstraction

- Physicist's point of view
- Chemist's point of view
- Biologist's point of view
- Sociologist's point of view
- Ecologist's point of view



Atom

The Physicist

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$$

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

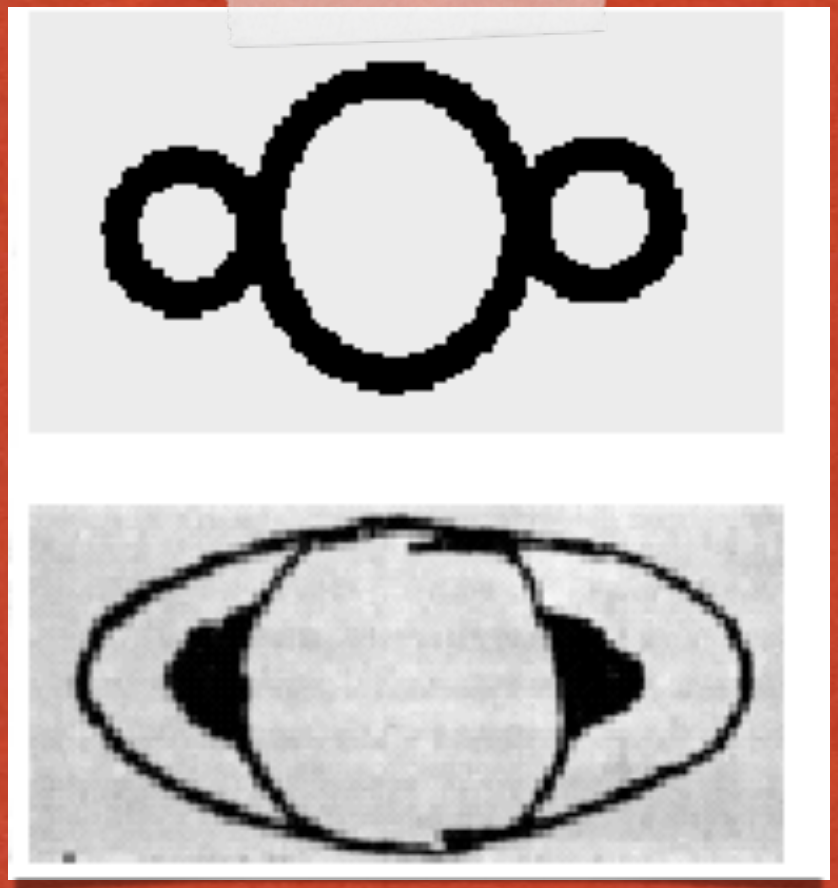
$$\mathcal{D}^{\mu\nu} = \frac{1}{\mu_0} g^{\mu\alpha} F_{\alpha\beta} g^{\beta\nu} \sqrt{-g}$$

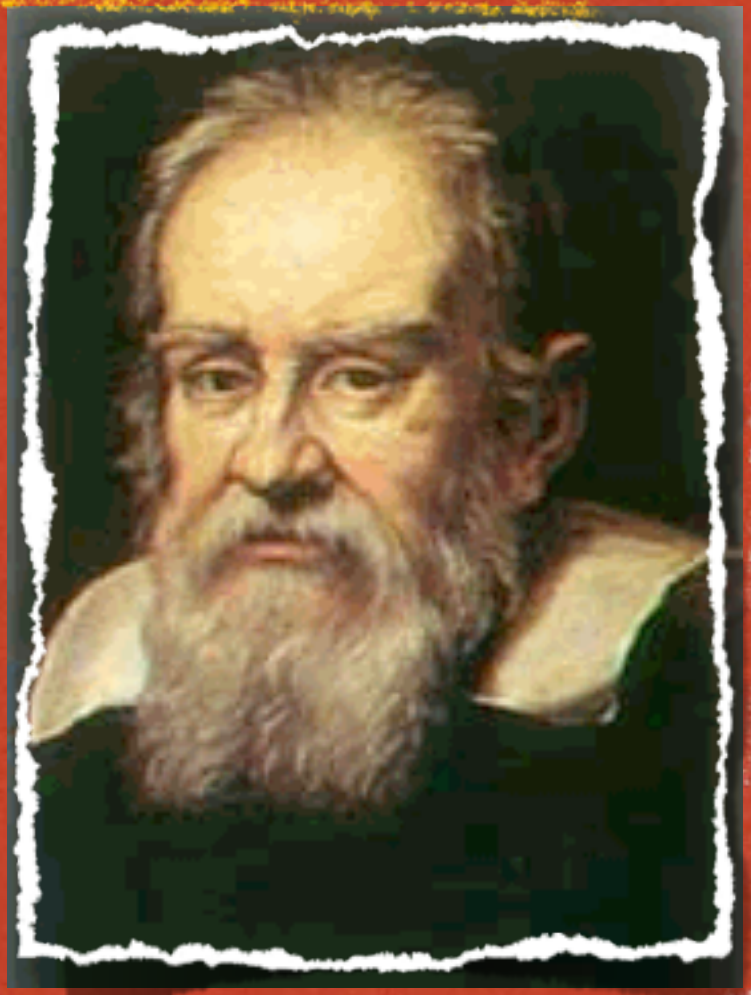
$$J^\mu = \partial_\nu \mathcal{D}^{\mu\nu}$$

...

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$

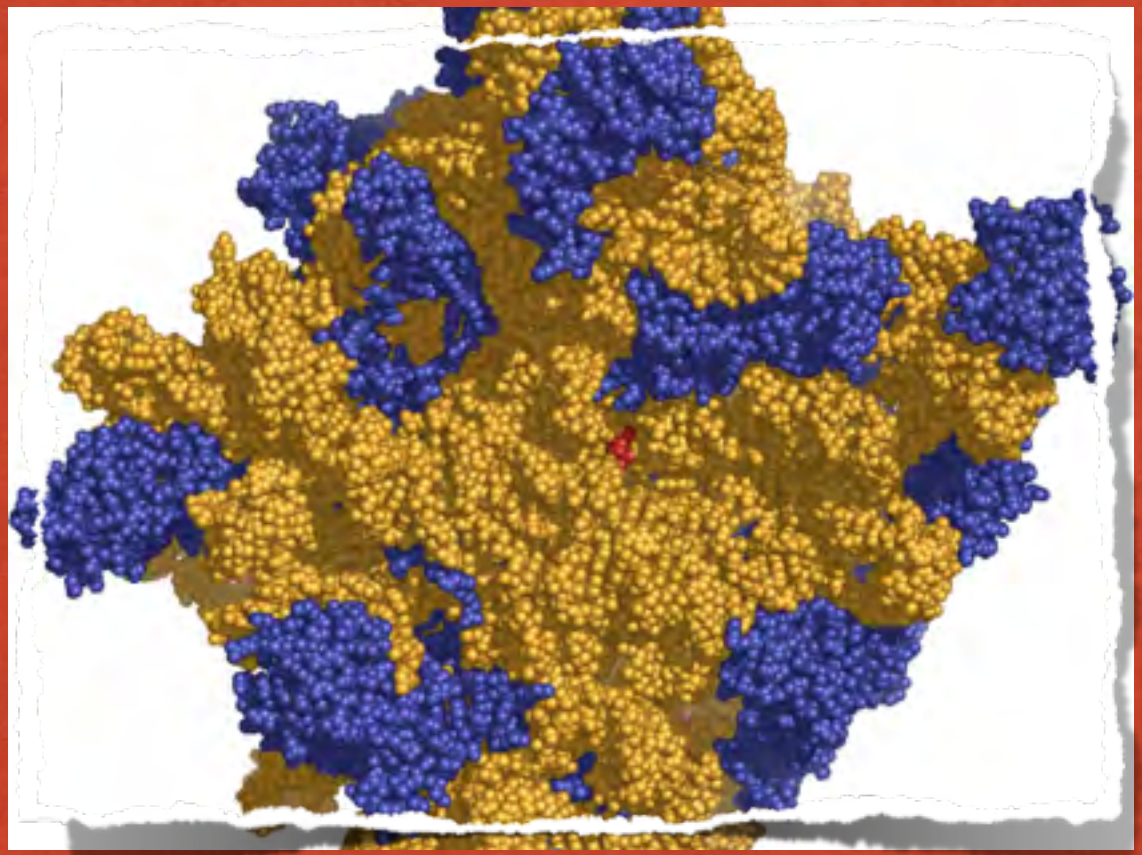
Galileo Galilei





Galileo Galilei

La filosofia è scritta in questo grandissimo libro che continuamente ci sta aperto innanzi a gli occhi (io dico l'universo) ma non si può intender se prima non s'impara a intender la lingua e conoscere i caratteri né quali è scritto. Egli è scritto in lingua matematica e i caratteri sono triangoli, cerchi, ed altre figure geometriche senza i quali mezi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro aberinto.



Molecule

The Biochemist

$$\frac{dX}{dt} = k(t) - \delta X + k_{off}C - k_{on}(p_{TOT} - C)X$$

$$\frac{dC}{dt} = -k_{off}C + k_{on}(p_{TOT} - C)X,$$

Cell

The Biologist

$$\partial_t \boldsymbol{q} = \underline{\underline{\boldsymbol{D}}} \nabla^2 \boldsymbol{q} + \boldsymbol{R}(\boldsymbol{q}),$$

...

Organism


11:32:24 hours
10 µm

Population

The Social Biologist

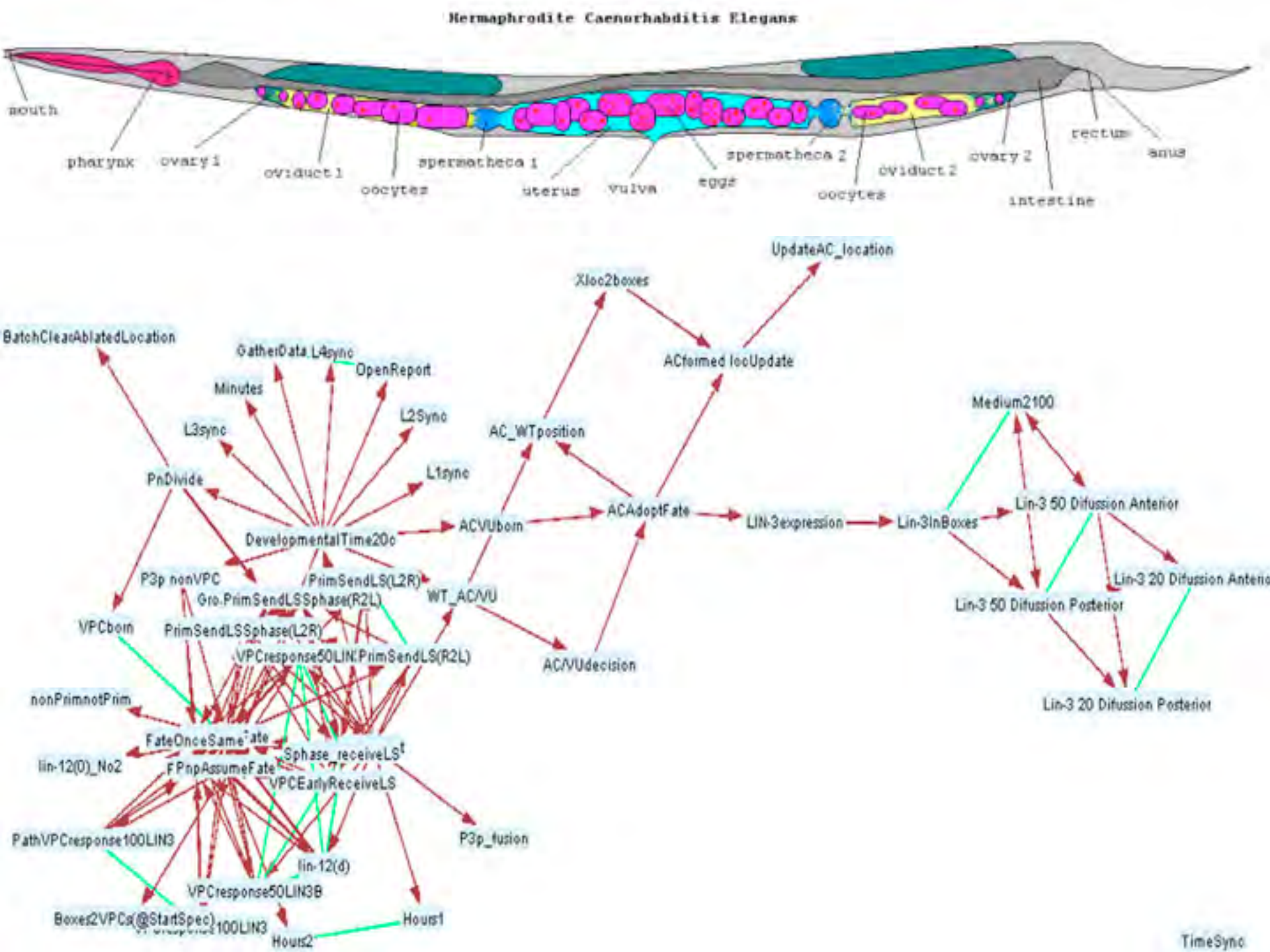
$$p(t) = \int_0^{K+t} dw \exp\left[-z \int_0^{w} du \frac{(1-e^{-u})}{u}\right]$$

Ecology

The Ecologist

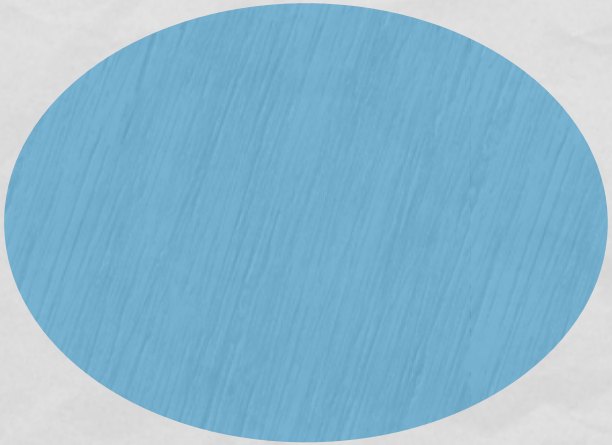
$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K(t)}\right)$$

Evolution

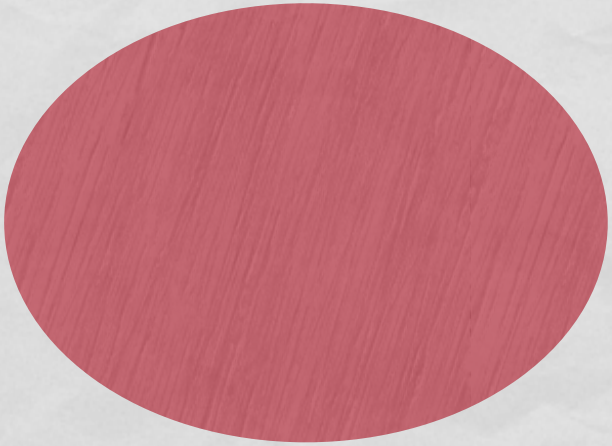

Hermaphrodite Caenorhabditis Elegans
mouth
pharynx
ovary 1
oviduct 1
oocytes
spermatheca 1
uterus
vulva
eggs
spermatheca 2
oocytes
oviduct 2
ovary 2
intestine
rectum
anus
UpdateAC_location
Xloc2boxes
ACformed locUpdate
BatchClearAblatedLocation
GatherData L4sync
OpenReport
Minutes
L2Sync
L3sync
L1sync
PnDivide
AC_WTposition
ACAdoptFate
LIN-3expression
Lin-3InBoxes
Medium2100
Lin-3 50 Difussion Anterior
Lin-3 20 Difussion Anterior
DevelopmentalTime20c
ACVUborn
P3p nonVPC
PrimSendLS(L2R)
Gro.PrimSendLSSphase(R2L)
WT_AC/VU
Lin-3 50 Difussion Posterior
VPCborn
PrimSendLSSphase(L2R)
VPCresponse50LIN.PrimSendLS(R2L)
AC/VUdecision
Lin-3 20 Difussion Posterior
nonPrimnotPrim
FateOnceSame Fate
Sphase_receiveLS
lin-12(0)_No2
FPnpAssumeFate
VPCEarlyReceiveLS
PathVPCresponse100LIN3
P3p_fusion
lin-12(d)
VPCresponse50LIN3B
Boxes2VPCs(@StartSpec)
Hours1
Hours2
TimeSync

Algorithms

An Evolving State



An Evolving State



An Evolving State

Discrete Evolution

Turing's Thesis

Turing machines
capture mechanical
human computation

Church-Turing Thesis

- Turing machines simulate (up to isomorphism) all effective models (under some representation).

Ada Lovelace

Many persons ... imagine that because [Babbage's Analytical Engine] give[s] its results in numerical notation, the nature of its processes must ... be arithmetical and numerical rather than algebraical and analytical.

This is an error.

Ada

The engine can arrange and combine its numerical quantities exactly as if they were letters or any other general symbols.

Turing - Post

- Look – at any of a fixed number of locations
- Draw – at any of those locations
- Move – one of a fixed number of "heads"

Emil Post

For full generality, a complete analysis would have to be made of all the possible ways in which the human mind could set up finite processes for generating sequences.

Generic Algorithms

Definitions

- Generic notions
  - Language-independent
  - Data-independent

Don Knuth (1966)

Algorithms are concepts which have existence apart from any programming language... Algorithms were present long before Turing et al. formulated them, just as the concept of the number "two" was in existence long before the writers of first grade textbooks and other mathematical logicians gave it a certain precise definition.

Euclid (c. -300)

Euclid's GCD algorithm was formulated geometrically: Find common measure for 2 lines.

Used repeated subtraction of the shorter segment from the longer.

Euclid's Antenaresis

- Finitely describable – in terms of basic compass operations

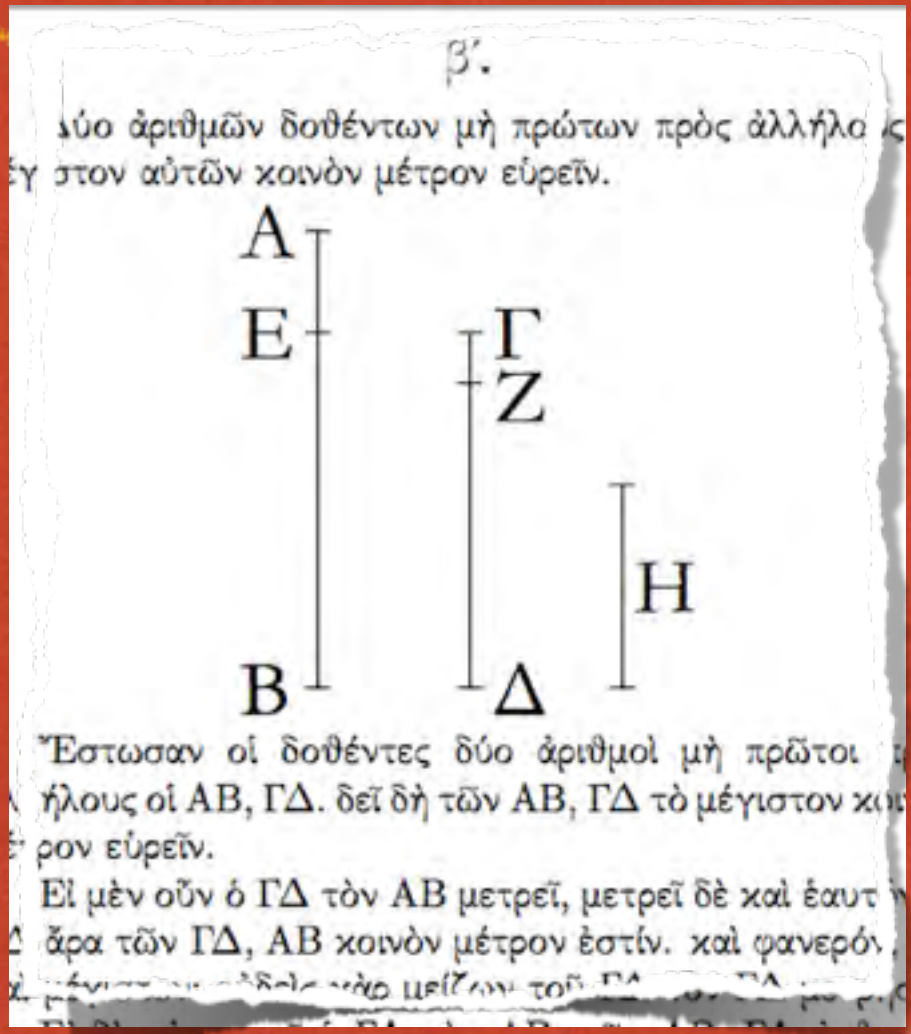
Antenaresis

Δύο ἀριθμῶν ἀνίσων ἐκκειμένων, ἀνθυφαιρουμένου δὲ ἀεὶ τοῦ ἐλάσσονος ἀπὸ τοῦ μείζονος, ἐὰν ὁ λειπόμενος μηδέποτε καταμετρῇ τὸν πρὸ ἑαυτοῦ, ἕως οὗ λειφθῇ μονάς, οἱ ἐξ ἀρχῆς ἀριθμοὶ πρῶτοι πρὸς ἀλλήλους ἔσονται

When two unequal numbers are set out, and the less is continually subtracted in turn from the greater, if the number which is left never measures the one before it until a unit is left, then the original numbers are relatively prime.

Euclid's Antenaresis

- Finitely describable – in terms of basic compass operations



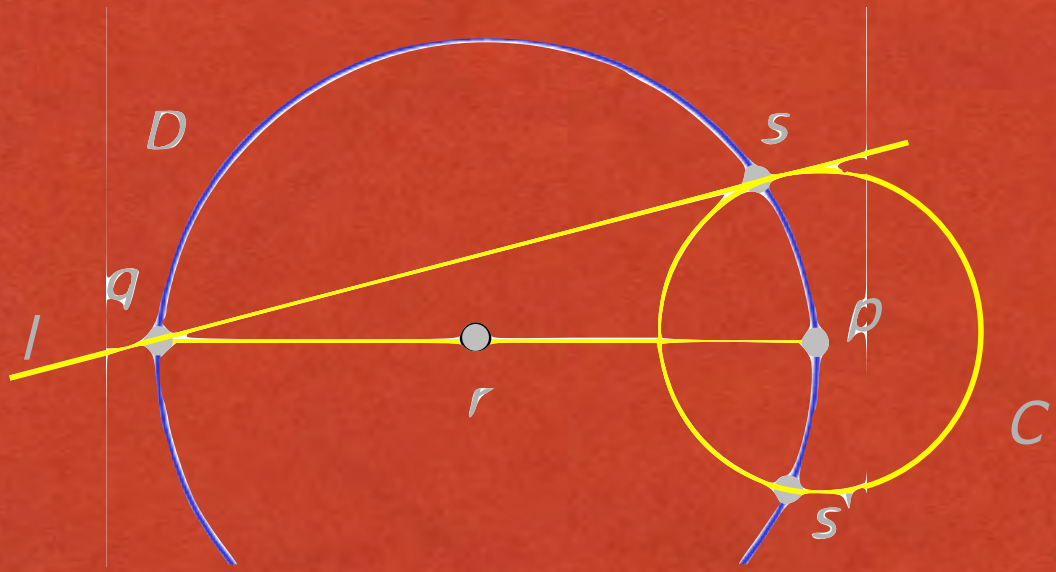
β'.

δύο ἀριθμῶν δοθέντων μὴ πρώτων πρὸς ἀλλήλα
ἐγ στον αὐτῶν κοινὸν μέτρον εὑρεῖν.

A
E
B
Γ
Z
Δ
H

Ἔστωσαν οἱ δοθέντες δύο ἀριθμοὶ μὴ πρῶτοι
ήλους οἱ AB, ΓΔ. δεῖ δὴ τῶν AB, ΓΔ τὸ μέγιστον κοι
ρον εὑρεῖν.
Εἰ μὲν οὖν ὁ ΓΔ τὸν AB μετρεῖ, μετρεῖ δὲ καὶ ἑαυτ
ἄρα τῶν ΓΔ, AB κοινὸν μέτρον ἐστίν. καὶ φανερόν

Euclid's Computer



If q ∉ C then

r := bisect(p,q)

D := Circle(r,q)

s :∈ C ∩ D

out := Line(q,s)

Unordered Domain

- Points
- Sets
- Membership, union, ...

A Neolithic Algorithm

Eve's Algorithm

- If something's left and it's my turn
  - Put one in my pile
  - Now it's his turn
- If something's left and it's his turn
  - Put one in his pile
  - Now it's my turn

Émile Borel

Les calculs qui peuvent être réellement effectues...

Je laisse intentionnellement de côté la plus ou moins grande longueur pratique des opérations l'essentiel est que chacune de ces opérations soit exécutable en un temps fini, par une méthode sûre et sans ambiguïté.

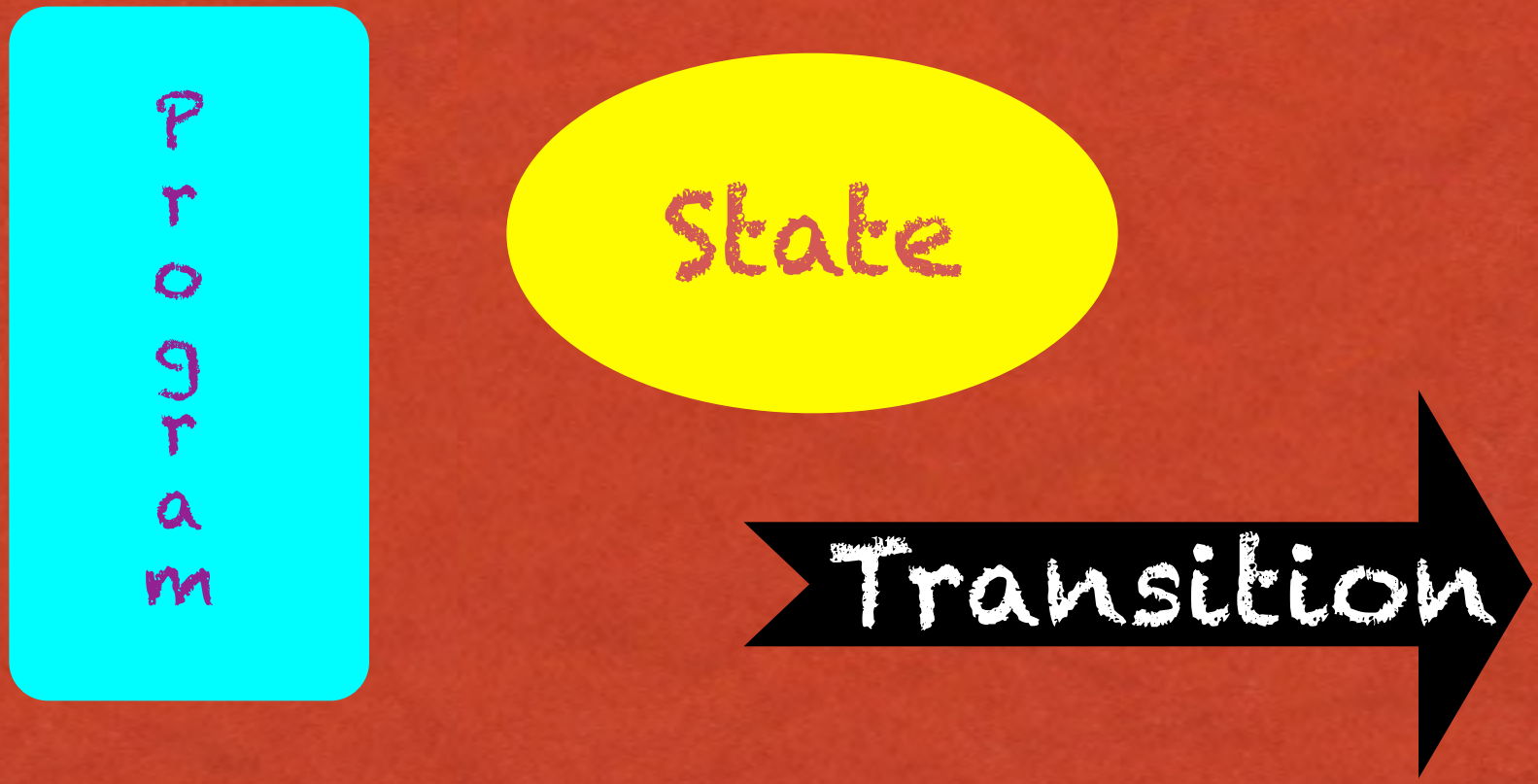
Knuth (1966)

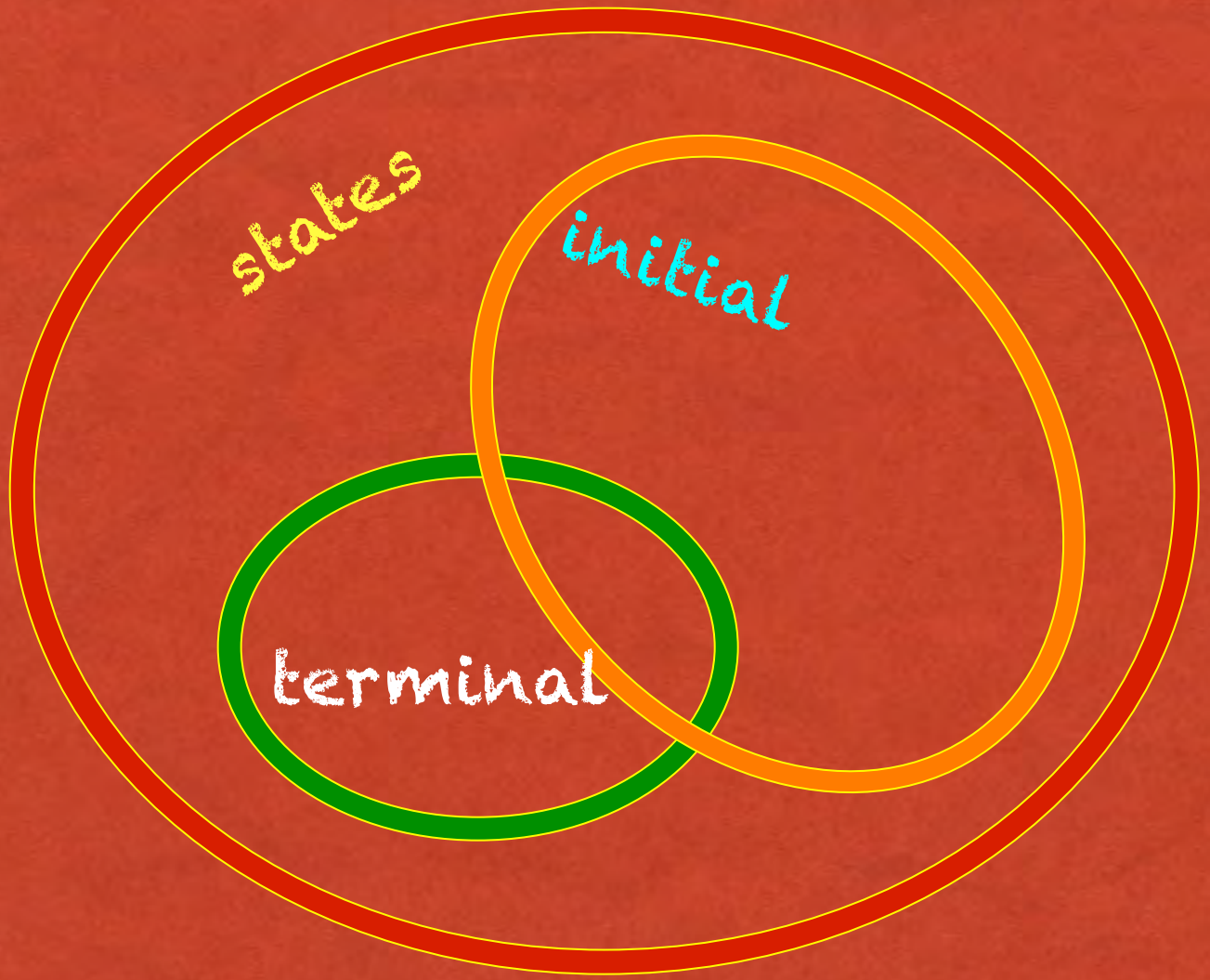
A computational method comprises a set of states...

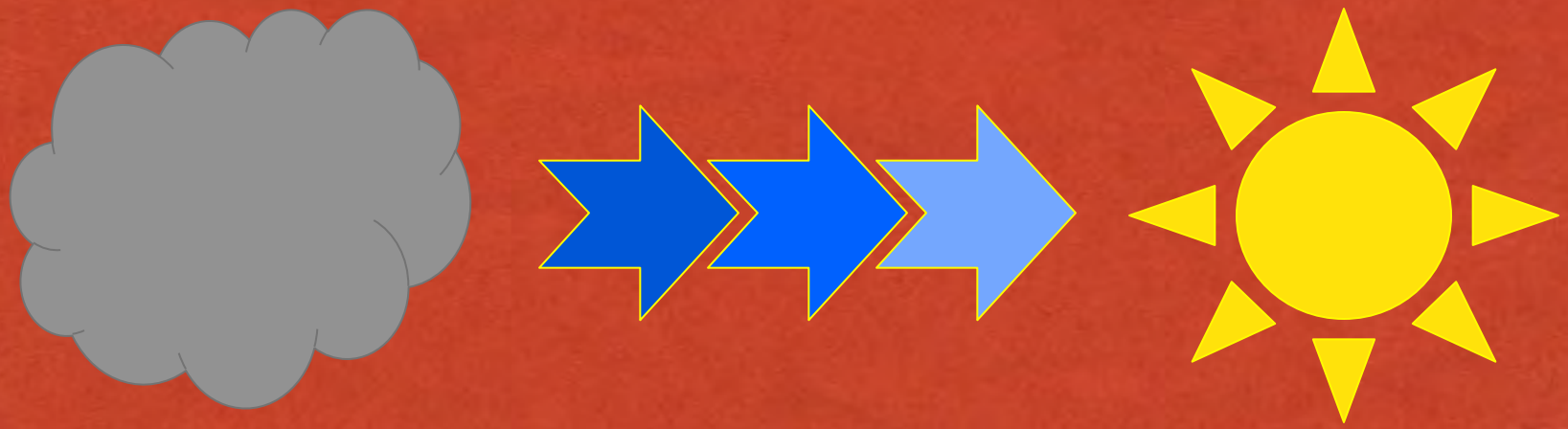
In this way we can divorce abstract algorithms from particular programs that represent them.

Transition System

Algorithmic System




states
initial
terminal



Joe Shoenfield

A method must be mechanical...

Methods that require insight are excluded.

Disallowed

Intermediate States

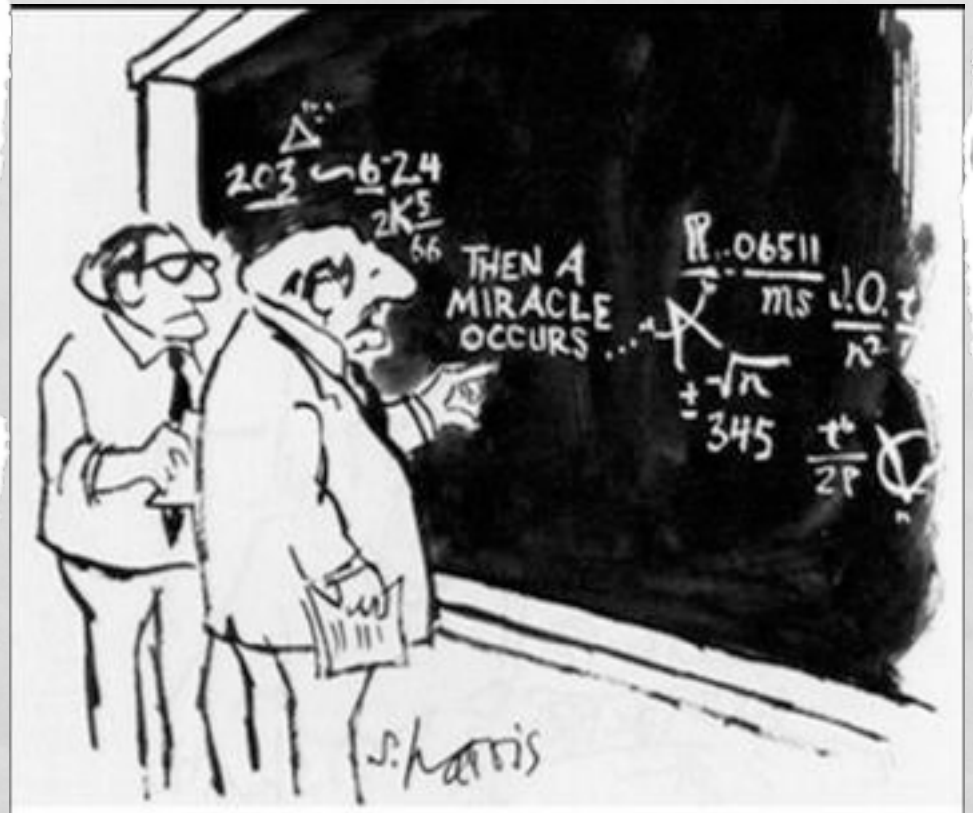
Ce genre de Peinture ... de pouvoir être interrompu quand on veut & repris de même

Paul Romain Chaperon,
Traité de la peinture au pastel (1788)

Hartley Rogers

- Roughly speaking, an algorithm is a clerical (i.e., deterministic, bookkeeping) procedure which can be applied to any of a certain class of symbolic inputs and which will eventually yield, for each such input, a corresponding symbolic output.

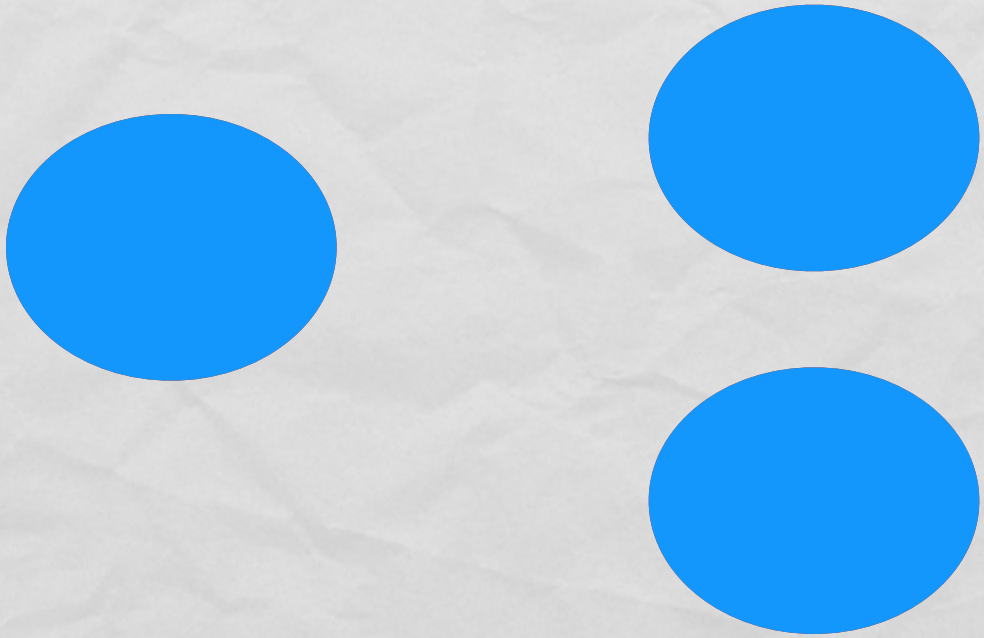
Mechanical

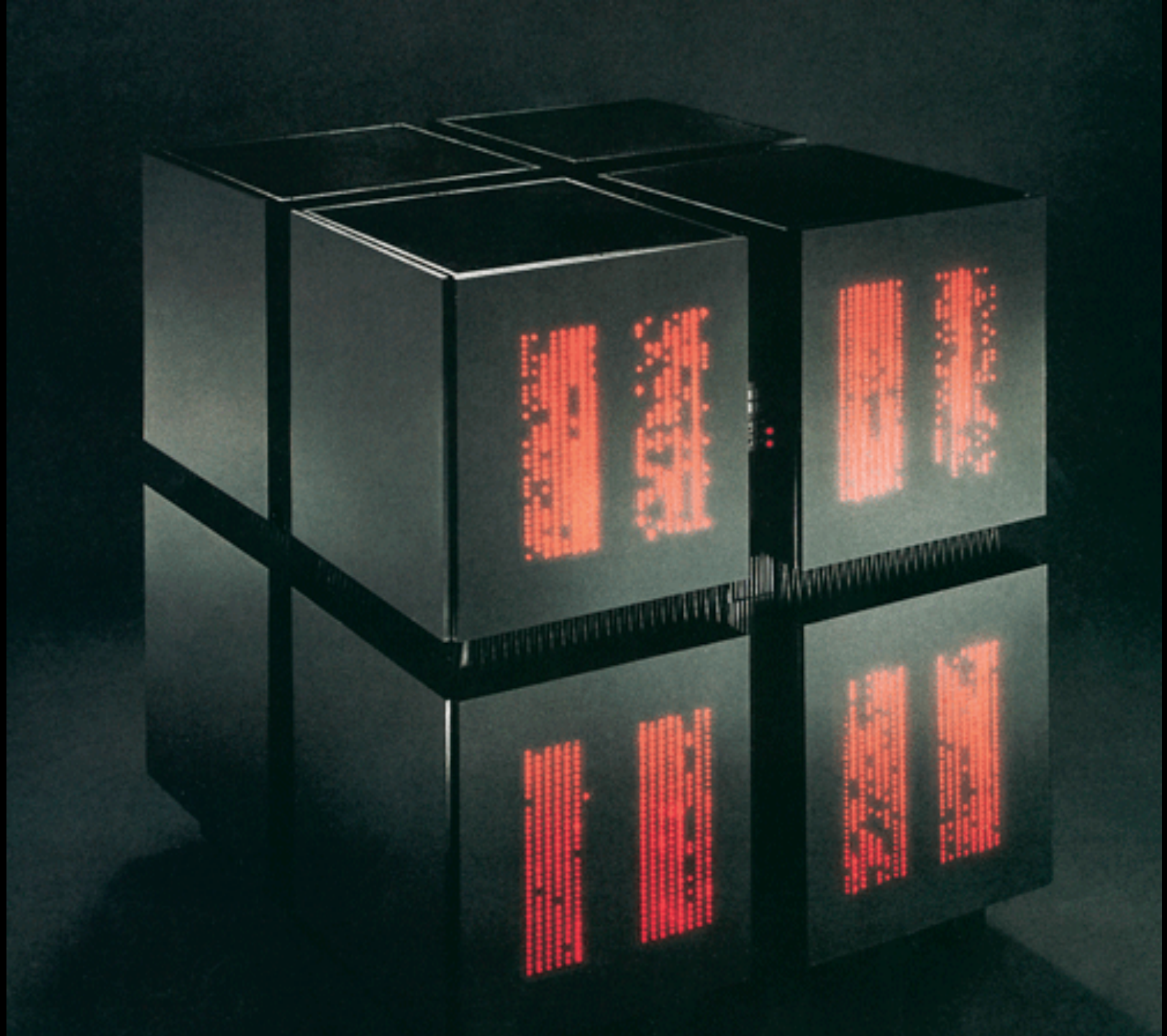


"I think you should be more explicit here in step two."

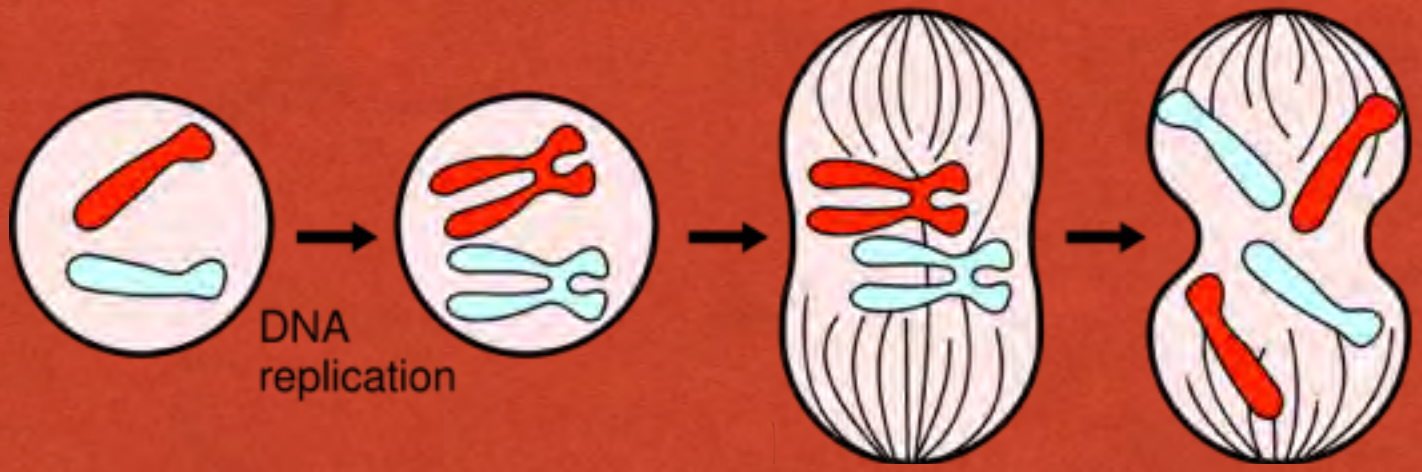
Parallel Algorithms

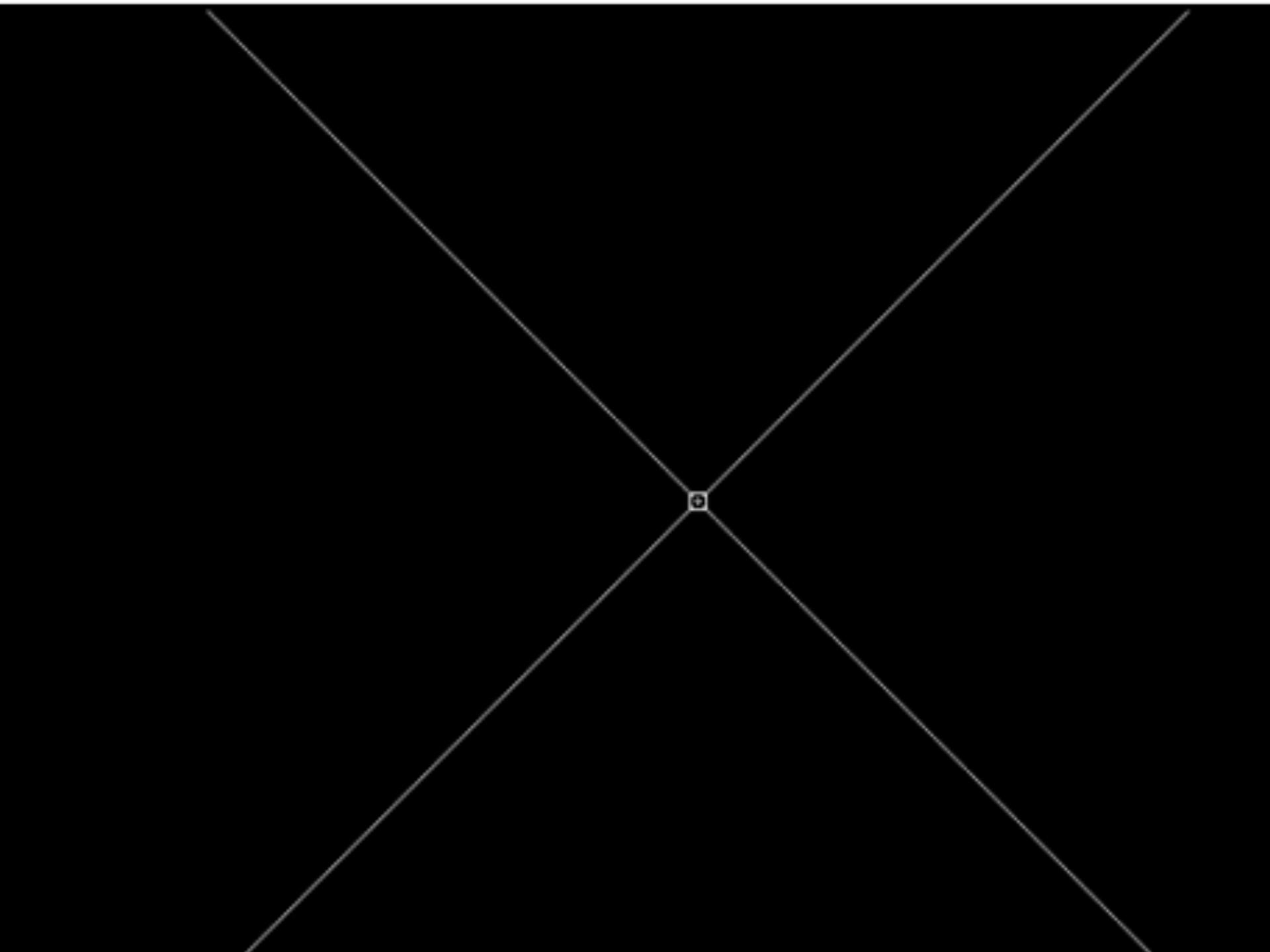
Parallel Evolution





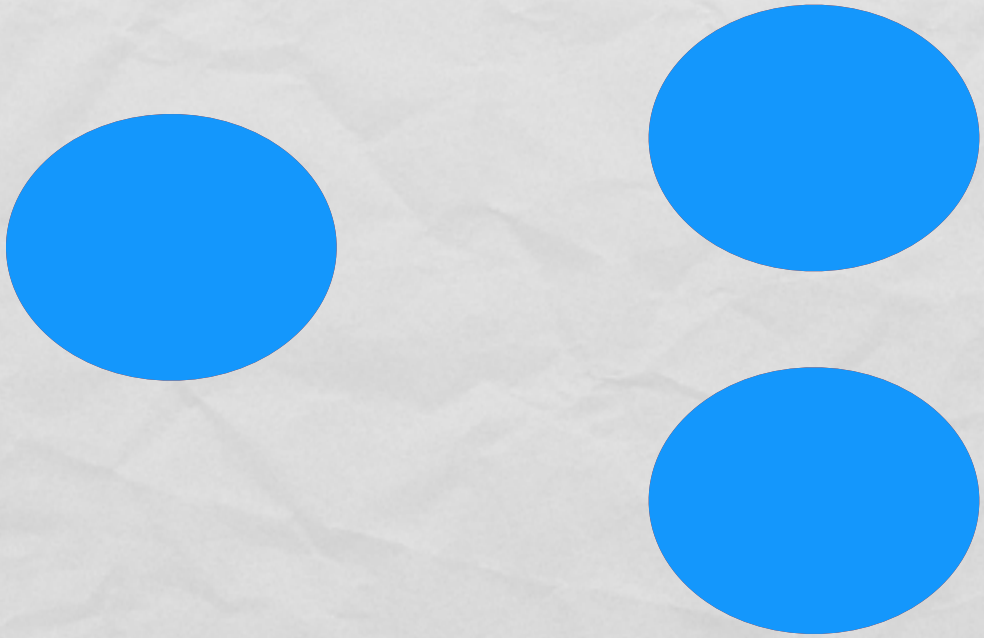
Cellular Evolution



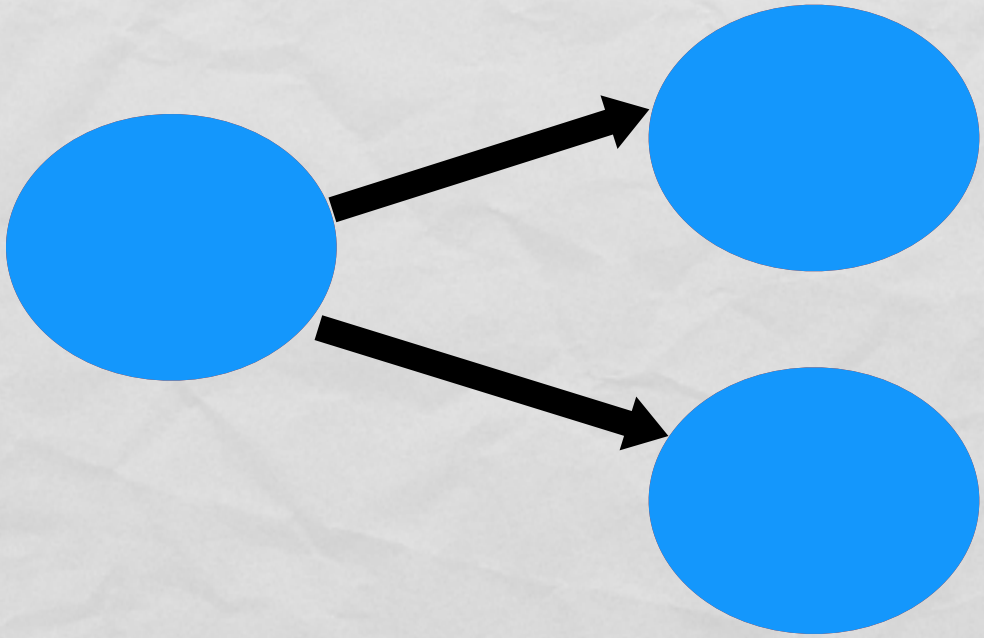


Concurrent Algorithms

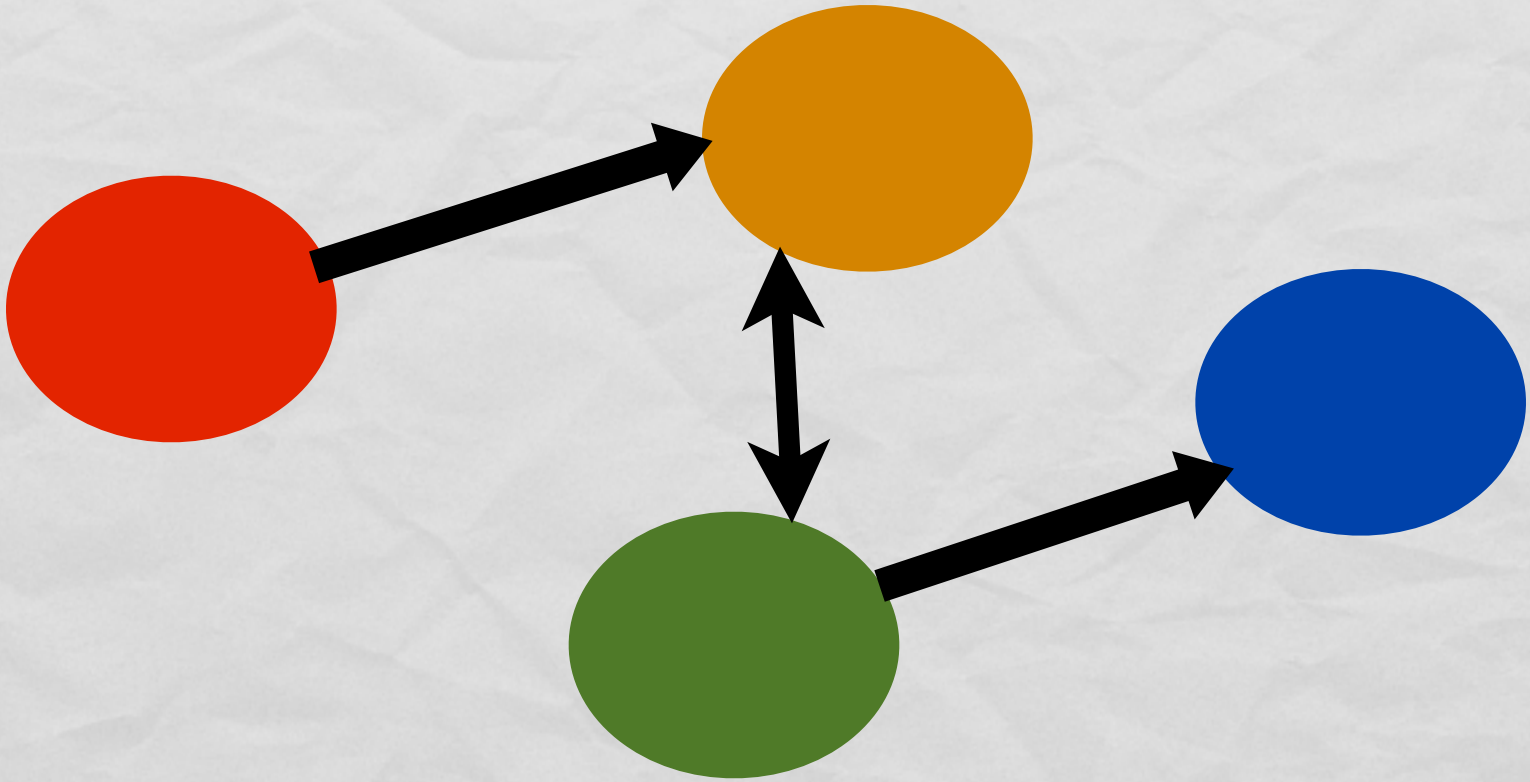
Concurrent Evolution



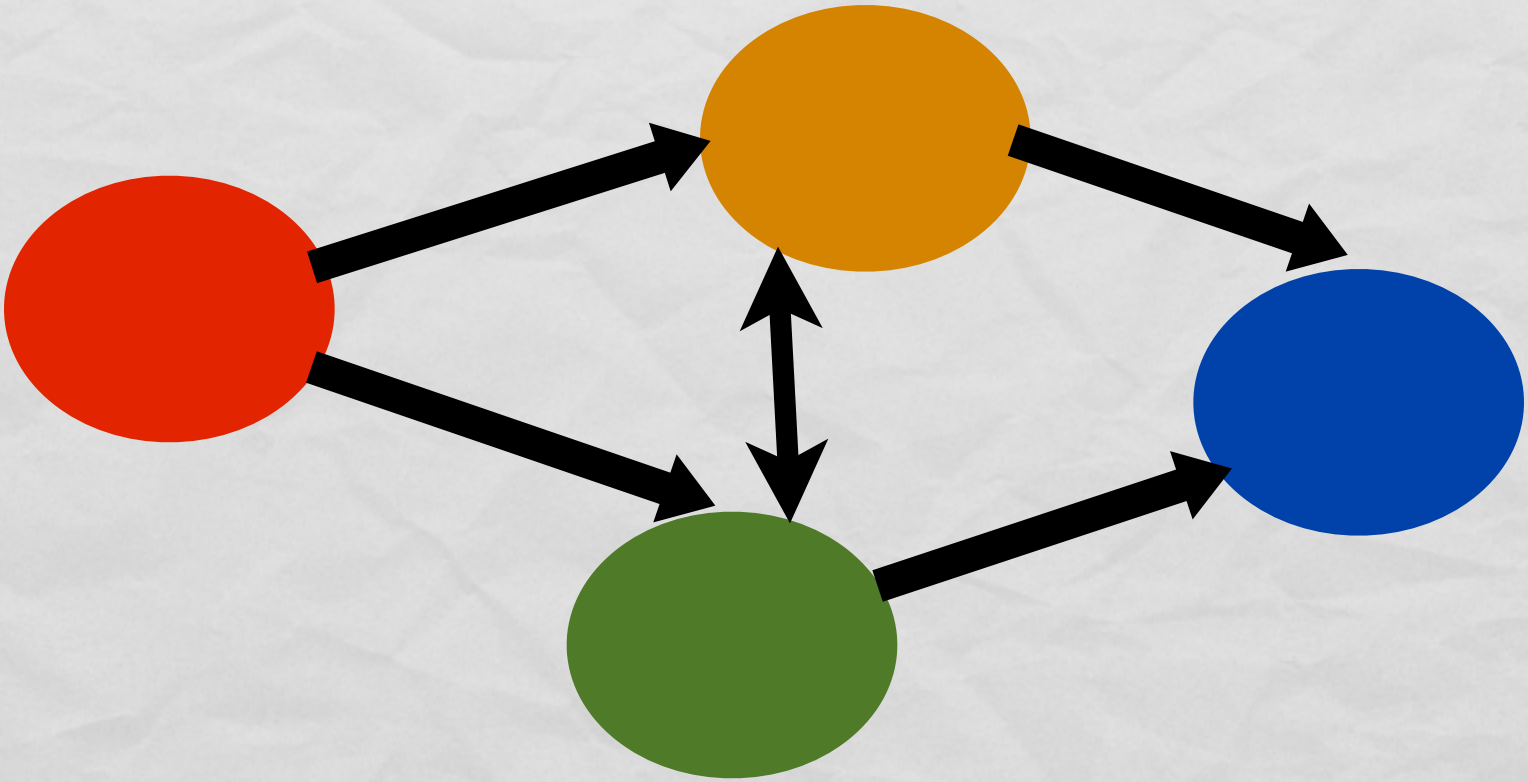
Interaction



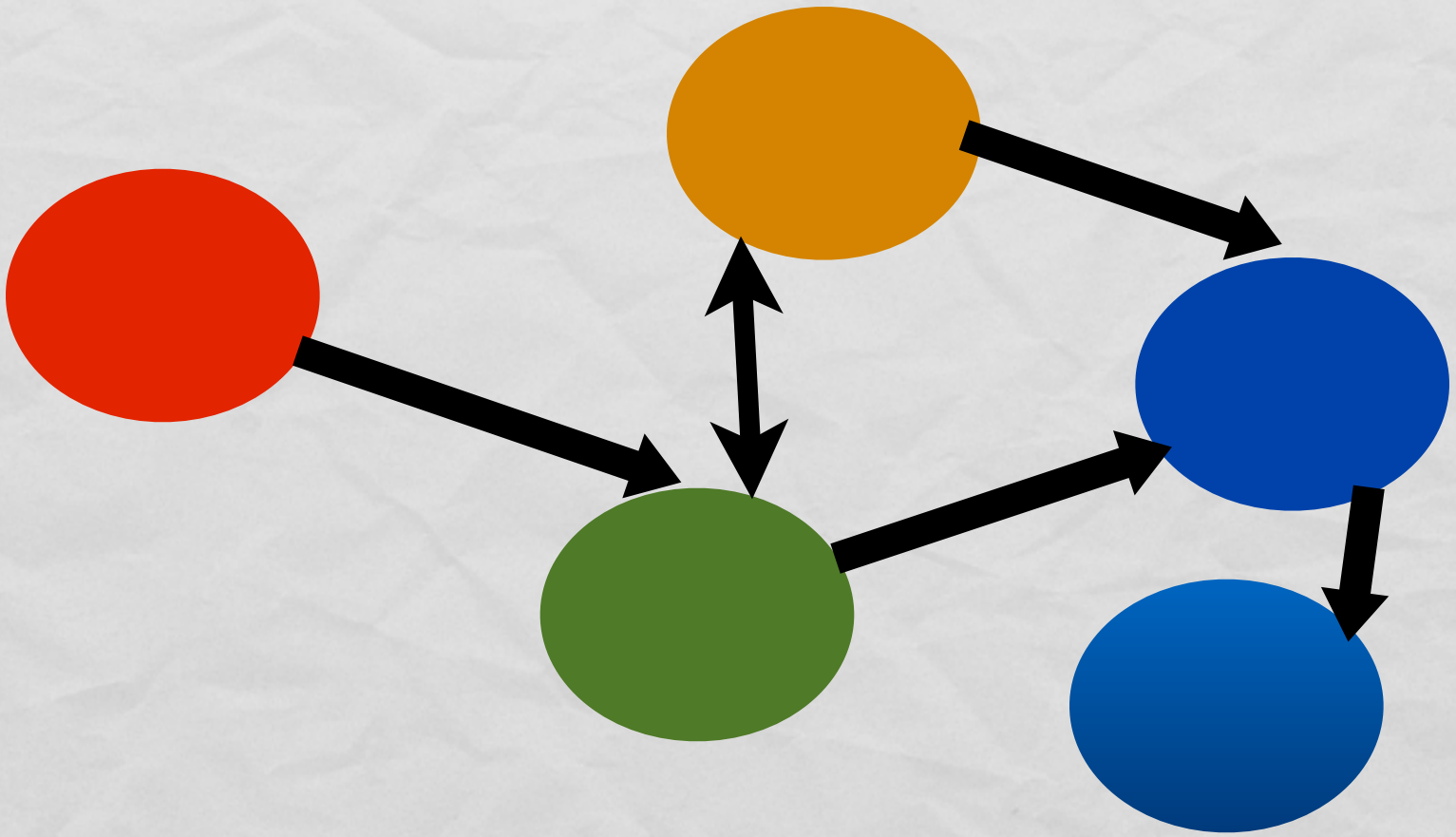
Evolving System



Evolving Topology



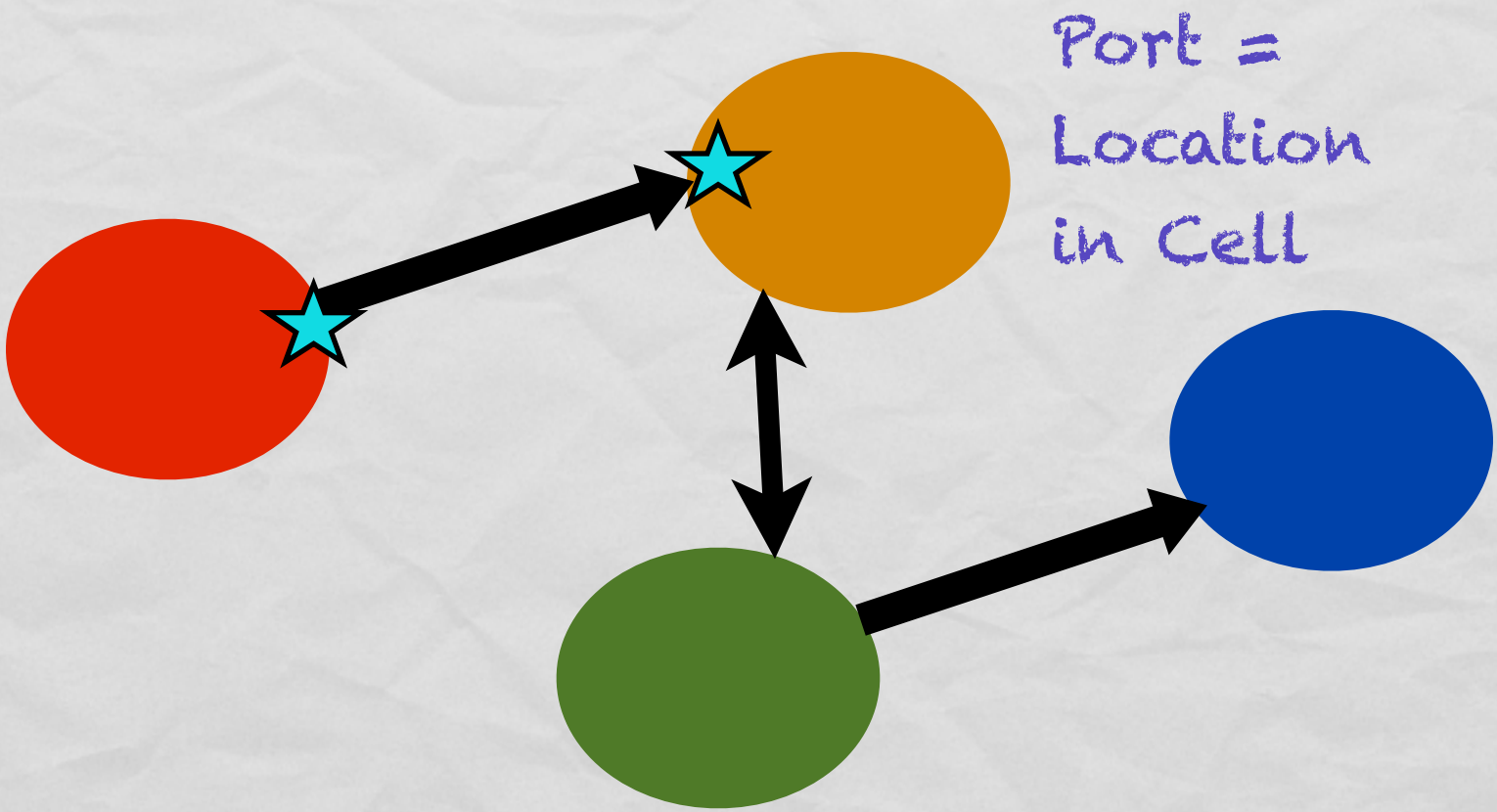
Childbirth



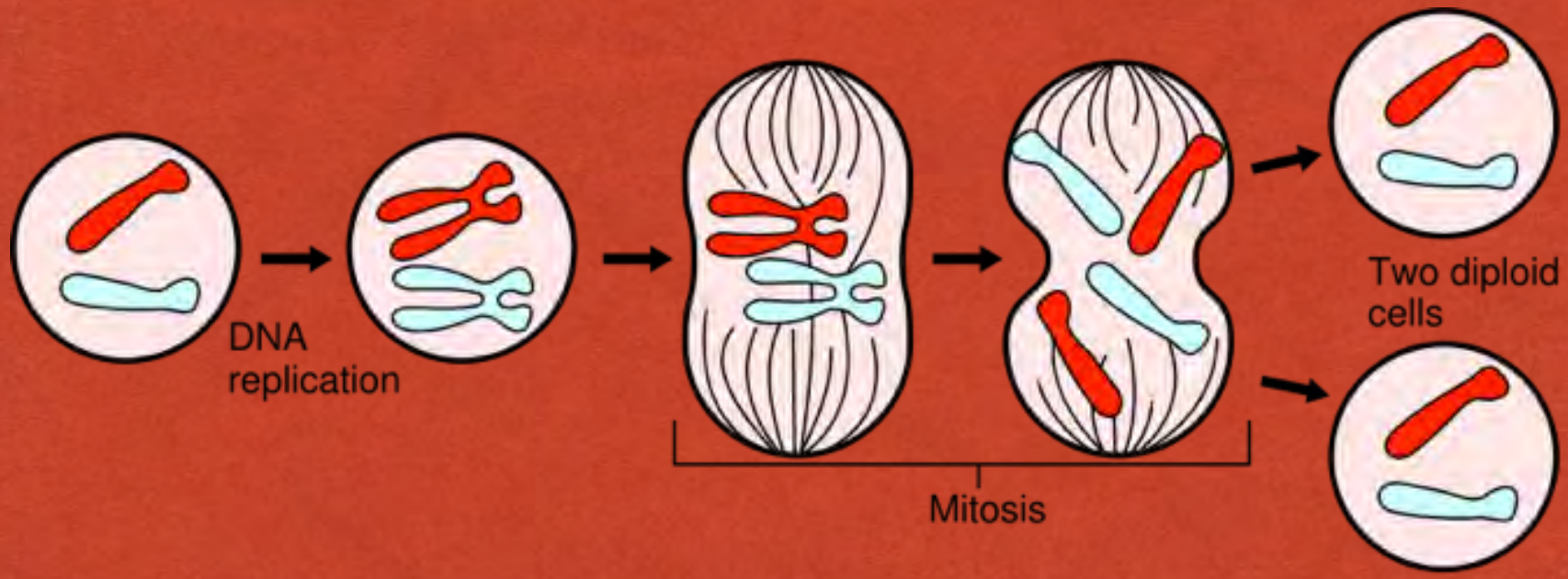
Concurrency

- Asynchronous message passing
- Changing topology
- Continuous-time processes

Evolving Connections

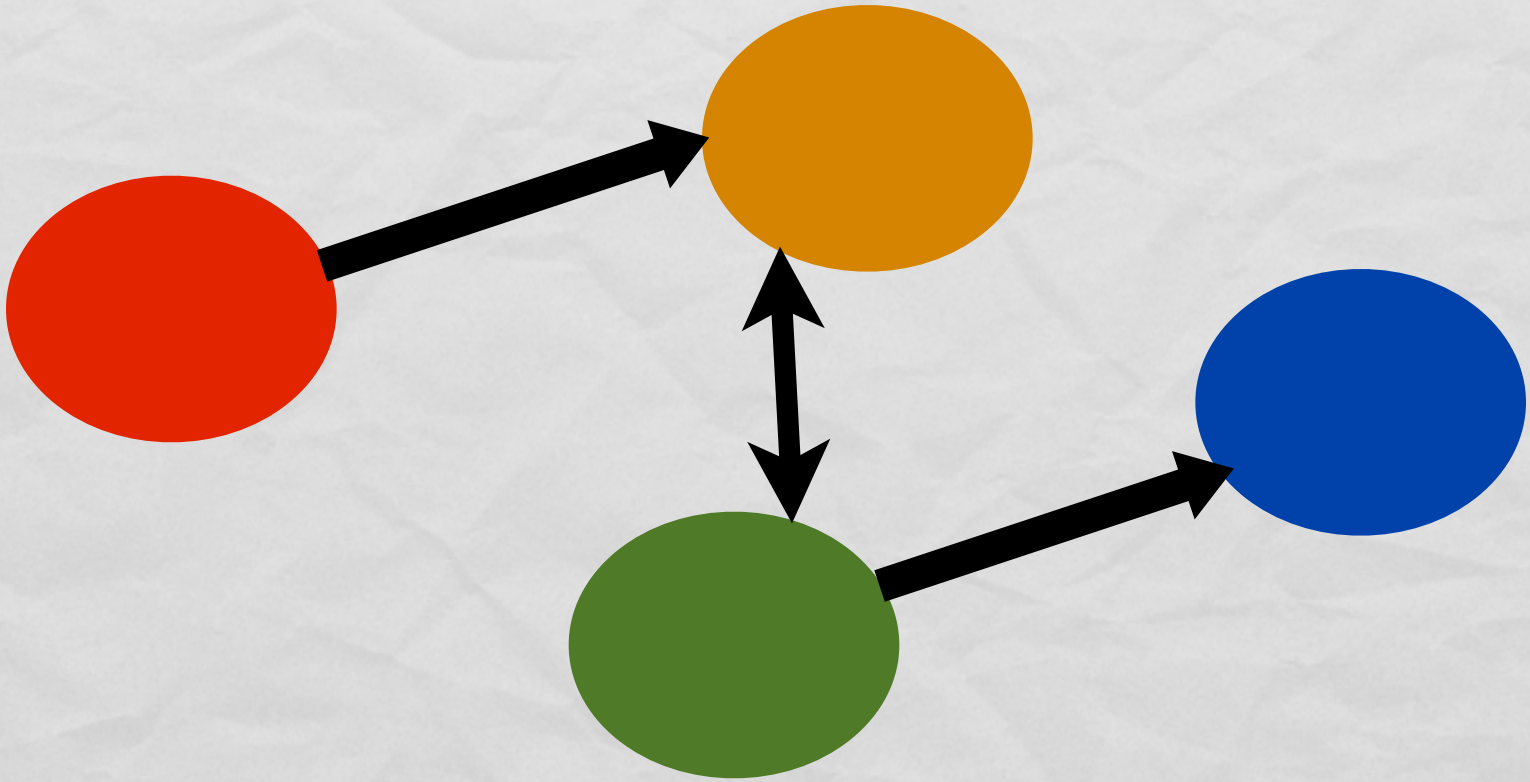


System Evolution

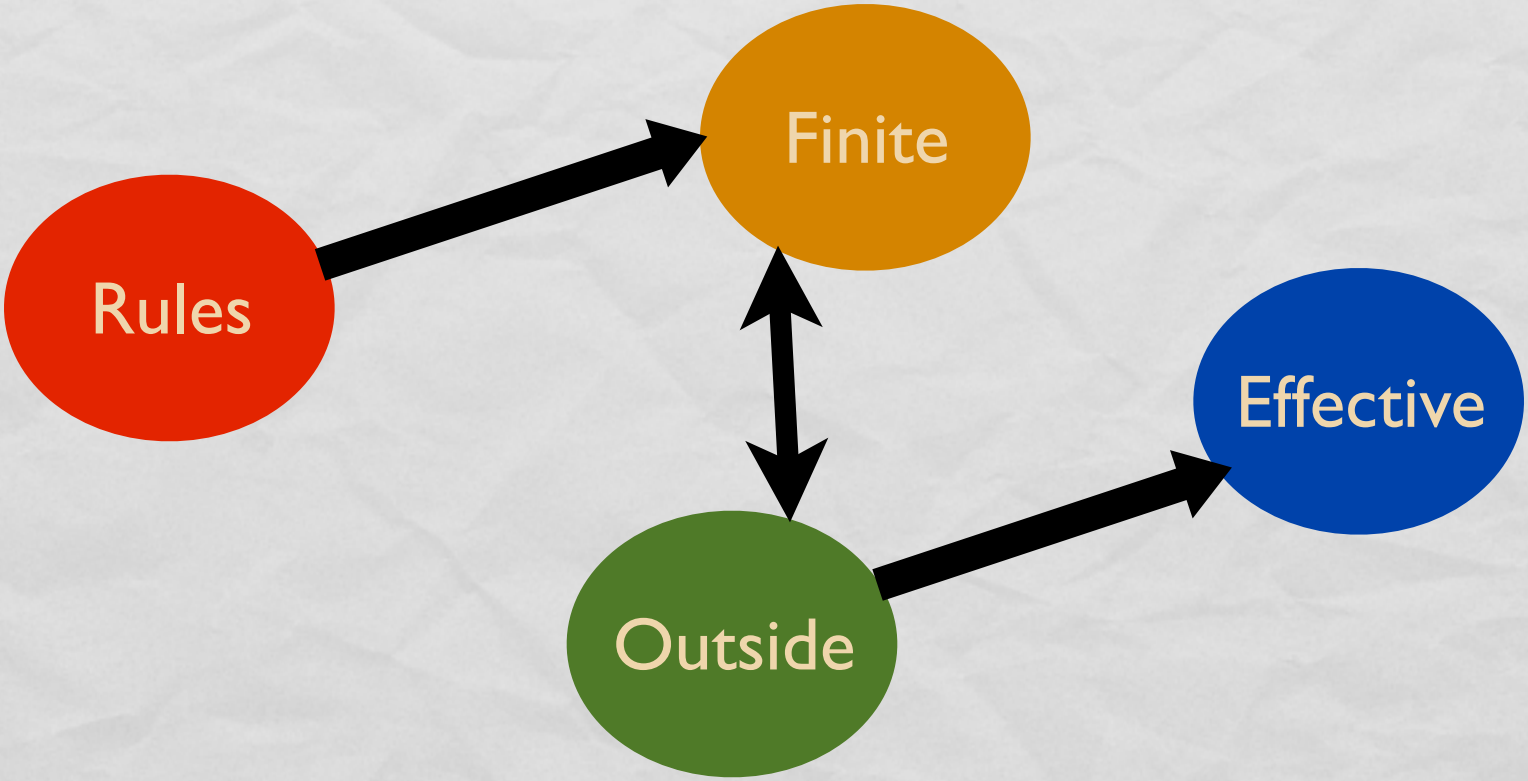


Hybrid Algorithms

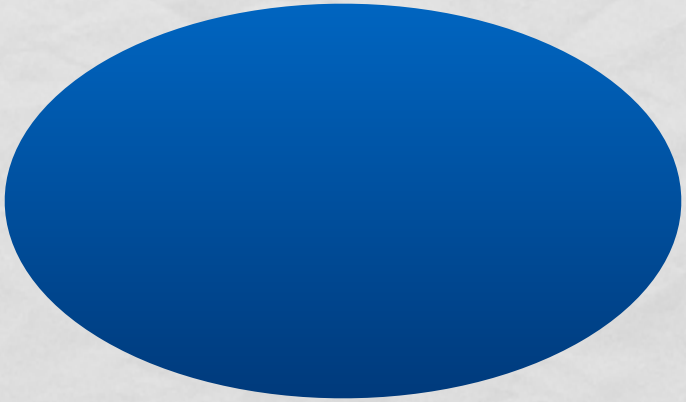
System



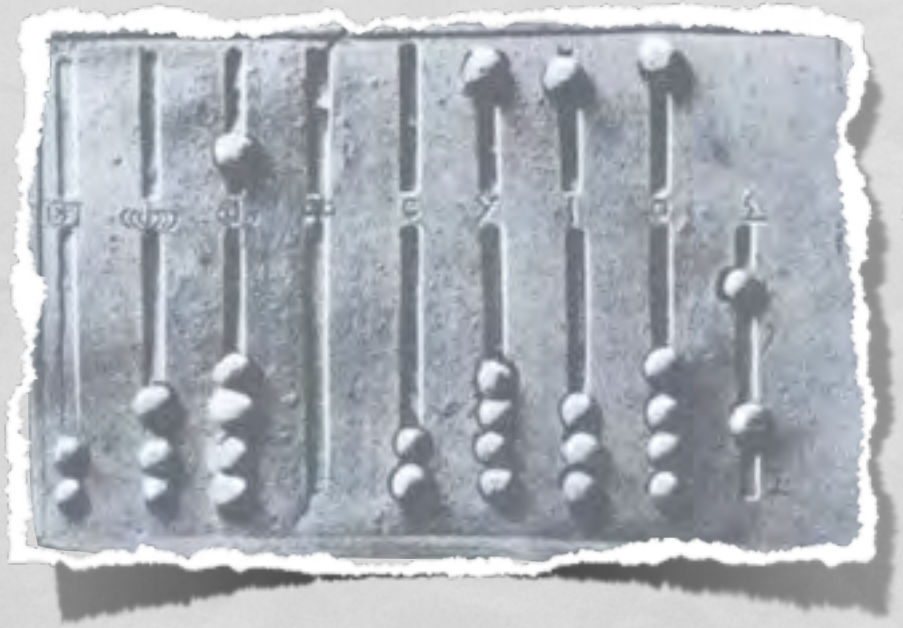
System



Continuous Evolution



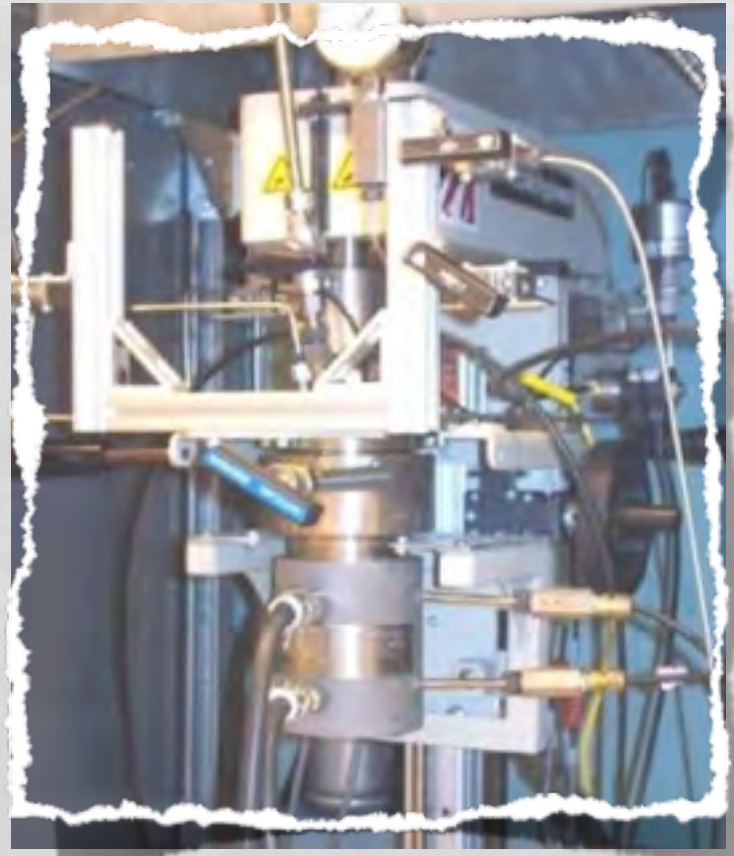
Digital vs. Analog

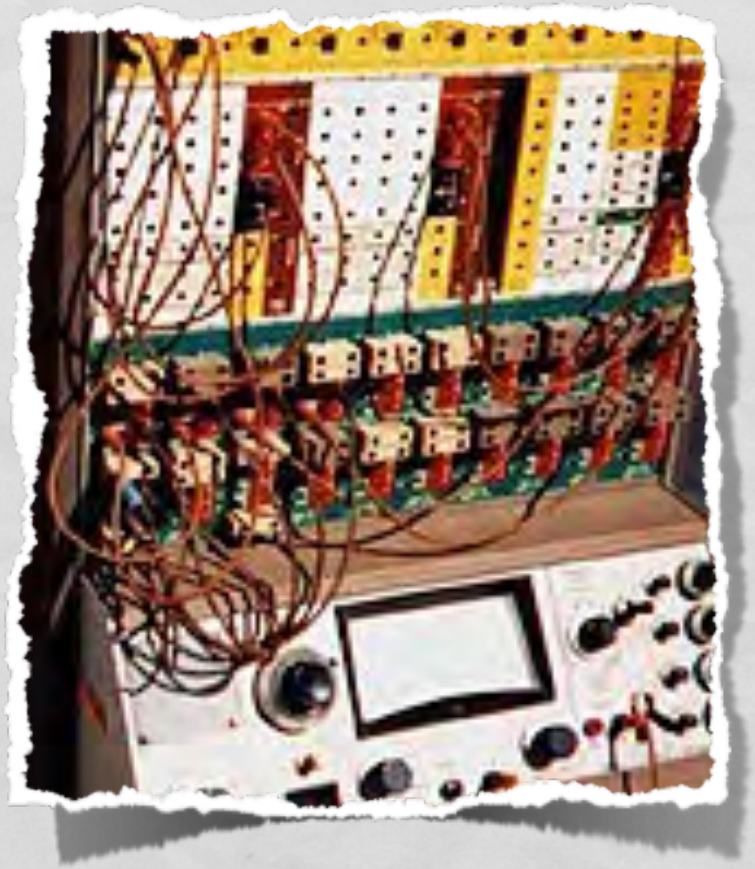


Analog Time

- State evolves as time progresses
- Time is dense or continuous

Compute by Analogy





isothermal semibatch bubble-column slurry reactor

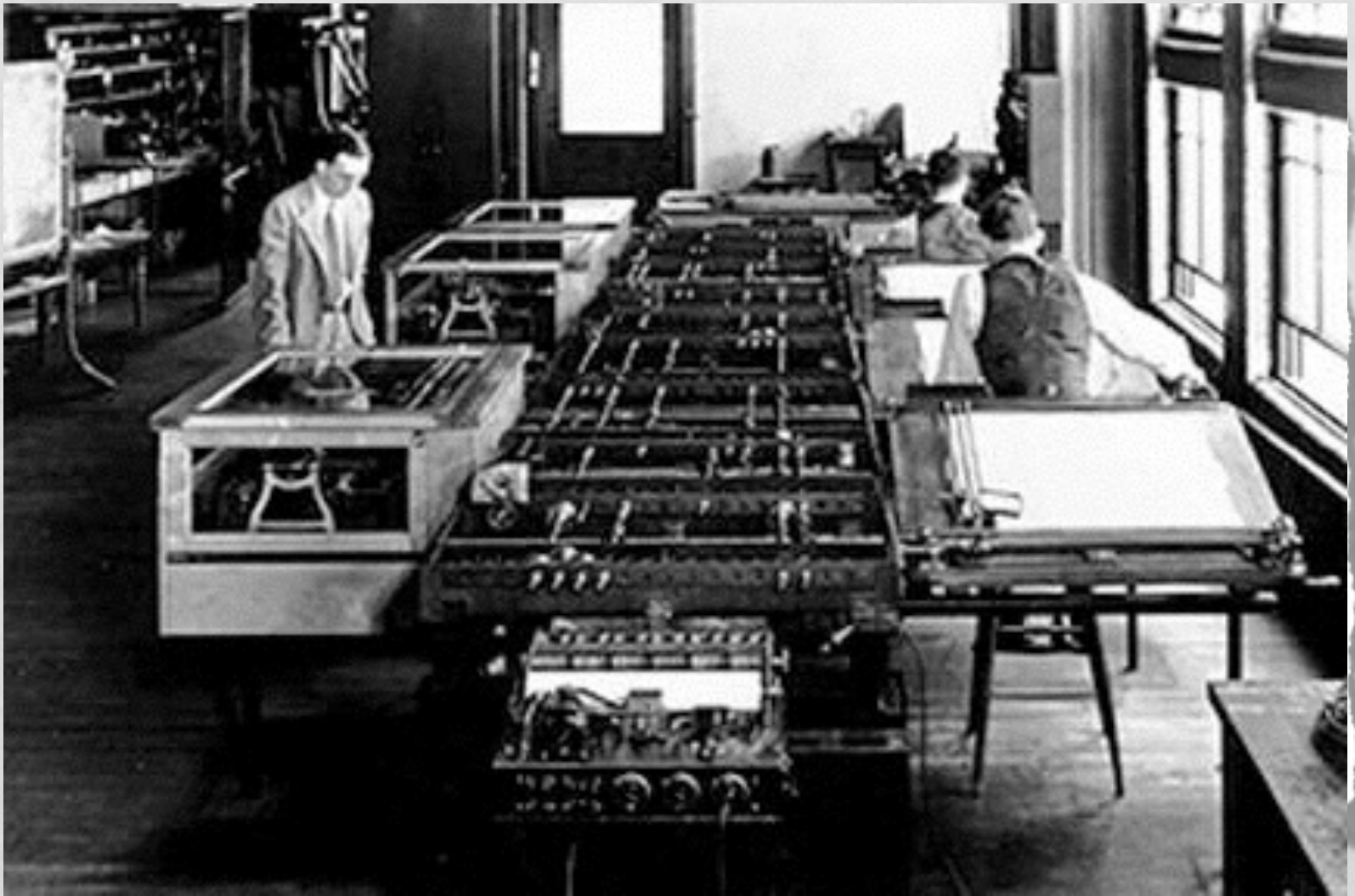
$$\frac{\partial c_1}{\partial \theta} = \frac{1}{P_{e}} \frac{\partial^2 c_1}{\partial Z^2} - \frac{\partial c_1}{\partial Z} - \frac{N}{m} (c_1 - c_2) \qquad (1)$$

$$\frac{\partial c_2}{\partial \theta} = \frac{1}{P_{el}} \frac{\partial^2 c_2}{\partial Z^2} + N \varepsilon (c_1 - c_2) - M f (c_2 - c_3) \qquad (2)$$

$$\frac{\partial c_3}{\partial \theta} = M f (c_2 - c_3) - Q f c_3 \qquad (3)$$

$$f = \exp(-P_p Z) \qquad (4)$$

Vannavar Bush's Differential Engine



Britt Phillips' Water Computer

Analog Program

Argonne National Laboratory

APPLIED MATHEMATICS DIVISION

ANALOG COMPUTER

5. POTENTIOMETER SETTINGS

PROBLEM NO. _____

DRAWING NO. _____

DATE _____

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
1		+ b volts	+10		1000		
2		- 1 volt	-1		0100		
21		ω/a	6.283		6283(10)		
22		ω/a	6.283		6283(10)		

If

b = 10

ω = 6.283

a = 1

A sine wave of 1 cps is generated

Hybrid Computers

Signals

- Function from interval of time to domain

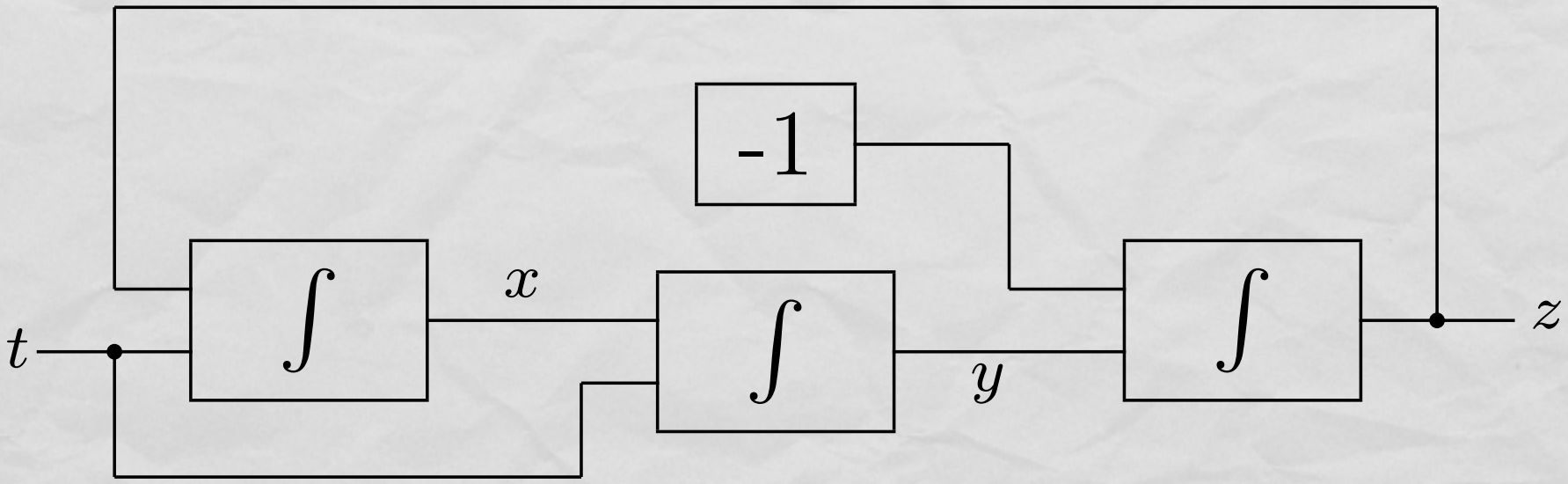
Flows

- Fixed dynamics over stretch of time
- If input wouldn't change, nothing would
- Equalities between critical terms maintained

Jumps

- Change of dynamics
- Requires conditionals

Implicit Flow



$x,y,z \ni \quad x' = z,\ y' = x,\ z' = -y$

Warren Smith's Newtonian System

- Can specify the 2D positions and velocities of n point-masses as rational numbers
- Cannot compute whether there's a collision within one second